

# THE DIRICHLET SPACE: CONNECTIONS BETWEEN OPERATOR THEORY, FUNCTION THEORY, AND COMPLEX ANALYSIS

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