Chapter 15
Assessing a Student’s Mathematical Knowledge by Way of Interview

DEBORAH LOEWENBERG BALL
WITH
BRANDON PEOPLES

Deborah Loewenberg Ball conducted the following interview with Brandon Peoples, a sixth grader, on March 8, 2004 at the first MSRI Workshop on Critical Issues in Mathematics Education.

This interview assessment of a student's mathematical understanding, conducted live in front of the assembled workshop participants, provides an immediate and vivid case of student thinking and exemplifies the interview assessment, an important mechanism for accessing student thinking.

1 Ball: [To audience] Brandon and I are going to pretend you’re not in the room, so I’m going to stop talking to you and we don’t actually care that you’re there, so, uh, goodbye. And uh, just be as quiet as you can because we actually want to do the work that we’re setting out to do here. So, okay?

[To Brandon] So, you know some of the stuff we were doing, before people came, we might go back to some of that just because some of it was interesting and we didn’t really finish talking about it. But I wanted to just start by, talking to you just a little bit about school. Do you remember when I called you the other day? We had a few minutes and I asked you a few things about your school and what you’re working on this year? So can we just go back to that, because we were – we didn’t have very much time that day. I was just wondering, like what you’re doing in math right now in school.

2 Brandon: Really, we’re learning about fractions.

3 Ball: Mm-hmm.
Brandon: And, umm, like – like we’re doing – we’re going on to like more – to more advanced of it instead of just – we’re just moving on up the line of the fr – of math.

Ball: So can you give me an example of something you’ve worked on, I don’t know, recently in fractions? Can you think of something you’ve been working on?

Time: 00:02:00

Brandon: Okay.

Ball: Do we have plenty of paper around? Do we have blank paper?

Brandon: Yeah.

Ball: Where is it? Oh, okay. No, you don’t have to change, I just wanted to make sure we had enough.

[Brandon starts to give his example.]

Ball: Okay, so what did you write?

[Brandon has written $\frac{1}{2} \div \frac{1}{2}$ on the overhead.]

Brandon: One-half divided by one-half.

Ball: And you’ve been working on this?

Brandon: Mm-hmm.

Ball: So, what have you learned to do with that?

Brandon: See, it’s the part – you have to – you have to multiply these two in, but instead you flip it upside down . . .

Ball: Uh-huh.

Brandon: So you, instead of one-half it’ll be two over one.

Ball: Okay, can you show me?

Brandon: Mm-hmm.

[Brandon writes $\frac{1}{2} \times \frac{2}{1}$ on the overhead.]

Ball: Okay. And after that there is another step that you take? Or what do you do next?

Brandon: You multiply –

Ball: Uh-huh.

Brandon: – and then you get your answer.

Ball: Can you do that?

Brandon: Mm-hmm.

[Brandon writes $\frac{2}{2} = 1$ on the overhead.]
Brandon: Which equals this to this or one whole.

Ball: Okay, umm, is there – did you make any pictures of that or anything like that or did you just kind of learn how to do it the way you just showed me?

Brandon: Well it’s – I learned different ways. I tried to come up with different strategies, but this was the better – this just adding and stuff is a better idea.

Ball: Well maybe we’ll talk about that a little bit more later. Do you remember – have any recollection or memory of when you first started to work on fractions at school? Like what grade it was?

Brandon: I think we started to – I think we started on in fifth grade –

Ball: Mm-hmm.

Brandon: because we didn’t – we didn’t learn – our teacher in fourth grade didn’t – we really didn’t learn fractions.

Ball: Mm-hmm. And what’s the new stuff this year? Like this I guess, right?

Brandon: Yeah, and, umm, this thing of my other math teacher, he calls it cross-canceling.

Ball: Mm-hmm . . . You mentioned that on the phone. What is that exactly?

Brandon: It’s like when instead of, like, going through all this reducing, instead it’s like two over – two over four times, umm, three sixths.

[Brandon writes $\frac{2}{4} \times \frac{3}{6}$]

Ball: Mm-hmm.

Brandon: See, you’ll want it – you’ll want to reduce that to maker it smaller instead so – and then you get – you come out with a fraction.

Ball: Mm-hmm. How do you do that?

Brandon: So I think you . . . And then you multiply.

[Brandon has written $\frac{2^1}{4} \times \frac{3}{\cancel{3}} = \frac{3}{4}$]

Ball: Okay, so can you explain what you just did?

Brandon: Mm-hmm. Umm, two-sixths can be reduced to? I mean oops.

[Brandon corrects what he has written: $\frac{2^1}{4} \times \frac{3}{\cancel{3}} = \frac{2}{4}$]

Brandon: Three – two sixths can be reduced to one-third, so I put a cross through the two, I put a one and cross this out and put a third. Multiplied one time – by three, and multiplied four times one.

Ball: But you have a three there now.

Brandon: Oh. Hold on, I think I didn’t do it right? three-twelfths, so it should be? Hmm.
Brandon: Let me see it’s three-twelfths but it can be reduced to I think it’s one-fourth or one-third. How did I get to . . . ? Wait, hold on. Something isn’t right.

Ball: Is that chair too high?

Brandon: Yeah, I think so . . .

Ball: So, do you want to leave this right now or do you want to work on this some more right now?

Brandon: We can work on it more.

Ball: You want to go back to the beginning then and tell me what you were trying to show me?

Brandon: I think I was trying to reduce it, but then I got the wrong answer. I reduced it, I got it, I add, I multiplied it, got an answer, but the answer could be reduced too.

Ball: What was the original problem that you wrote down?

Brandon: Was two-fourths times three-sixths.

Ball: Uh-huh. And do you – do you already have a sense of what you think the answer’s supposed to be to that?

Brandon: Yeah.

Ball: What is it you think it’s supposed to be?

Brandon: Well, I thought it was three-fourths, but it wasn’t real, so I, it was . . .

Ball: Why did you think it would be three-fourths?

Brandon: Because, umm, I put a one instead of a three.

Ball: Mm-hmm.

Brandon: I thought because, umm, two-sixths was supposed to be reduced to one-third, so –

Ball: Uh-huh.

Brandon: – but can re – it’s reduced – when it’s reduced to one third.

Ball: But you wrote one fourth. Do you mean one-fourth or one-third?

Brandon: One fourth. Yeah.

Ball: One-fourth. So now you’re no – which answer are you saying is correct? The three-fourths you originally wrote or the one-fourth you just wrote?

Brandon: One-fourth.
Ball: Okay. Maybe – let’s save this one and maybe – maybe if there’s some time when we’ve done some other work we’ll come back to it. Is that okay? ’Cause right now we’re just trying to get a sense of the different things you’ve been working on.

Brandon: Mm-hmm.

Ball: Do you ever use fractions any place besides in school?

Brandon: Mm-hmm, not really.

Ball: Does anybody? Do you know anybody who uses fractions anyplace other than in school?

Brandon: Mm-mm.

Ball: Uh-huh. So, why do you think you learn about fractions?

Time: 00:08:23

Brandon: I think so it’s like – it’s kinda – it kinda takes, umm – it’s kinda in relation to percentages.

Ball: Mm-hmm.

Brandon: Because like – because like one-fourth would be to – in percentage, it would be 25 percent.

Ball: Mm-hmm.

Brandon: And in, umm, decimals it would be zero-point-two-five

Ball: Mm-hmm.

Brandon: So it’s – I think it’s one way we can tell, umm, how much is something.

Ball: Okay.

Brandon: Or – or like how much, or what is – what percentage of it is.

Ball: Okay.

Brandon: Like – it’s like different things contain different things. So like one part of something would be a percent – a percentage or a fraction. Same thing.

Ball: Okay. Umm, I think we’re going to shift over and do some other problems and questions and things like that and I just wanted to remind you of something I told you before we started today which is anything, you’ve kind of already doing it, but anything I ask you, you can draw, you can write, you can use words. Any – anything that you’ve ever used before you can do, or anything that you think you can use – the main thing I’m trying to understand is how you’re actually thinking and what you’re thinking about. So even if I don’t, you know, suggest using these blocks, or I don’t suggest using a
drawing, you’re totally free to do that. You can use the board, we can use this thing. Okay, do you remember when I was talking about that?

Brandon: Mm-hmm.

Ball: And, um, it’s really not a test, I’m actually just trying to do some mathematics problems with you and see the ways that you think about them, kind of like you’ve already started to do. So you’re doing exactly what I was hoping you would do. And the more you can tell me about kind of how you’re thinking and why you’re thinking certain things, the more helpful that will be in what I’m trying to learn about watching you work. Okay?

Brandon: Mm-hmm.

Ball: So, umm, I think I’ll start with, umm – can you hand me one of those green pieces of paper? And maybe we’ll need clean paper to write on too, so can I have another one?

Brandon: Mm-hmm.

Ball: Thanks. Okay. Can you fold that piece of paper in half?

[Brandon folds paper in half.]

Okay. So can you just explain how you knew what – what to do when I asked you that? Like, how did you decide how to do it?

Brandon: Well, umm, you take it like this and then draw down the line so – I know that it would be half ‘cause it’s only two –

Ball: Mm-hmm.

Brandon: – it’s only two uh, two major things. So like ‘cause if it was like – if it was like this, then it would be in fourths [referencing his folded paper]. But since it’s – I only see two sides then I know it’s just half. They’re both halves.

Ball: If I asked a little kid to do that, like a five year old, do you think they would be able to do it too? Or do you think they might make a mistake when they did it?

Brandon: I think they would get, like, how to fold it in half, but I don’t think they could explain how would it be half.

Ball: Can you think of any mistakes a little kid might make? In folding it?

‘Cause you did it right and I’m curious if you can think of anything a little kid might do that would be a mistake that you didn’t make.

Brandon: They would probably – they would probably fold it a different way –

Ball: Mm-hmm.

Brandon: – ‘cause they might not know what a half is, or what like, what would – what would this be.
103 Ball: Like, what if they did something like this? Can you picture a little kid doing that?

[Ball folds a sheet of paper incorrectly: One side is obviously larger than the other.]

104 Brandon: Yeah, 'cause –

105 Ball: Why would they do that?

106 Brandon: 'Umm, probably 'cause I think they probably might not know what half is. They might just fold it. They might just fold it in their way.

107 Ball: Mm-hmm.

108 Brandon: Instead of like, er, fold it not like – not like this but like one side would be bigger and one would be smaller.

109 Ball: So another thing you – you – you were trying to do what to make the parts be the same?

110 Brandon: Mm-hmm.

111 Ball: Okay [referencing the folded paper]. So can you write down how much of the piece of paper is on this side, or can you just tell me how much that is? Just this amount of the paper. How much of the paper is that?

112 Brandon: Wh – half or, umm, that’s like fifty percent?

113 Ball: Can you write those down?

114 Brandon: Okay.

115 Ball: You can use this paper to write with.

[Brandon writes 50% on the left side of the paper.]

116 Ball: Okay. And what’s the other thing?

117 Brandon: 50 percent.

[Brandon writes 50% on the right side of the paper.]

118 Ball: Okay. And you also said half. How would you write half?

119 Brandon: Half. Umm…

[Brandon writes on the right and left side of the paper.]

120 Ball: Okay. Is there any other way you could write it?

121 Brandon: Yeah.

122 Ball: Oh I see, you’re writing it for each side, right? Is that what you’re doing?

123 Brandon: Mm-hmm.

[Brandon writes 0.50 on the right and left side of the paper.]

124 Ball: Okay. You mentioned – A minute ago you mentioned that you could fold it into fourths. Can you show me that one? How you’d do that?
[Brandon folds the same paper into fourths.]

Ball: Okay, so what did you do to that?

Brandon: Umm, it was already like this from the half, so I folded that into – into right here –

Ball: Mm-hmm.

Brandon: – and turned it into fourths.

Ball: Why did it turn into fourths?

Brandon: Cause since it – it already had a line right here from the – from half, so if I fold it in this one it’ll have another line right here and that separates them into fourths.

Ball: Okay. So what – how would you – how could you explain – how could you write down what amount that each of those portions of the paper are? Do you want that pen? Can you put it up here? [Referring to the projector.]

Brandon: Oh, sure.

[Brandon moves paper up to projector. He writes 0.25 and \(\frac{1}{4}\) on the top left quarter of the paper.]

Or umm?

[Brandon writes 25\% on the top left quarter of the paper.]

Ball: Okay. Now when you think about, umm, the paper you just had – this one – are there any other fractions you could use to represent how much? Actually we should – let’s fold this one. I think I should’ve just let you do it on the one you had. Okay, are there any other fractions you can use to write down how much this part of the paper is? What else could you write?

Brandon: Umm, you could write other – You could write, umm, one number and what would be half of it.

Ball: Like what?

Brandon: Like, umm, say for all of this could be – like, all of this could be thirty-eight, but, like only – only nineteen of it – this is nineteen and this is nineteen.

Ball: So how would you write that fraction?

Brandon: Ummm.

[Brandon writes \(\frac{19}{38}\) on the right and left side of the paper.]

Ball: Okay, and – go ahead. Okay, and is there another one you can do?

Brandon: Hmm. I’m thinking of one.
Brandon: 'Cause, umm, wait? No, actually no, it’s thirty-seven. Thirty-seven.

Brandon corrects both $\frac{32}{74}$ fractions to $\frac{37}{74}$.

Ball: So how – how did you do this one?

Brandon: Umm. 'Cause this – all of this could be equal to – is seventy-four, but half of it would be thirty-seven.

Ball: Okay. Can you do similar things with one-fourth? What would be an example of something like that? Can you write another fraction for one-fourth?

Brandon: Okay.

Ball: How would you do it with fourths? Just a minute. Okay. [adjusts paper on the projector]. Okay, so what did you do this time?

[Using the paper folded into fourths, Brandon has written $\frac{2}{8}$ on the overhead.]

Brandon: Umm, two-eighths 'cause it takes four – it take four twos to equal eight, so two would be 25 percent –

Ball: Okay.

Brandon: – or one-fourth.

Ball: Okay. Let’s go to a different… can you get another piece of paper? Let’s do one more of these paper-folding things. Do you have a blank one over there somewhere?

Brandon: Yeah.

Ball: Could you fold the piece of paper in thirds?

Brandon: Thirds? I think so.

Ball: I’m sorry that chair is too tall. Do you remember how we adjusted it?

Brandon: Yeah…
Ball: Oh, I know. It’s that lever over on your left – on your right. I think if you – If you pull on that… Are you uncomfortable? Okay.

Brandon folds paper in thirds. He estimates the first fold, and sees that the next fold will not come out even with the paper’s edge. So he then adjusts the first fold to correct before finishing.

Ball: You can put it on the desk if you want. Okay, so how did you do this one?

Brandon: Umm… One – I put one right here and then folded it.

Ball: It seemed like this was a little bit more difficult than folding the half or the fourth. Why was that?

Time: 00:19:00

Brandon: ’Cause you – you can’t exactly – See it – with thirds it’s like you try to fold it right – try to fold it right – in right where you, umm, put the crease into it so you know that all the sides would be even, but with fourths all you have to do is just fold it like this –

Ball: Mm-hmm.

Brandon: – and – but half is just like this.

Ball: What do you get if you fold it like you did – remember when you had the half and you folded it this way? What do you get if you do that this time?

Brandon: You get sixths?

Ball: Okay. Can you write, umm, like [we just worked – we’ll spend a minute on this] can you write what fraction would you put for that portion?

Brandon: I think this would be…

Ball: Thirty-three and a third is…

Brandon: Mm-hmm.

Ball: Thirty-three and a third is…

Brandon: So there – this is…

Ball: Because when – if you just – ’Cause all this is – I think it’s one-hundred, but when you put thirty three into it, it’s – it’s – it – it’ll go on and on with threes –

Ball: Mm-hmm.

Brandon: – so what would be better is to just put one-third.

Ball: Okay. Are there any other fractions you can write for this?

Brandon: I don’t think… Mm-mm.

Ball: No? How much would you write for this much of it; for two of the sections?

Brandon: So that’ll be six, so… Umm. I’m not sure.
Brandon: Umm.
[Brandon folds the paper into sixths as he is working on finding an answer.]
Brandon: I think sixteen.
Ball: You think what?
Brandon: Sixteen.
Ball: Sixteen?
Brandon: Mm-hmm.
Ball: How would you write it?
Brandon: 'Cause – since this is thirty-three, then I think it would be sixteen
because I just thought that if fifteen – it’s – it’s almost, umm, half of thirty-
three, so if this was sixteen . . . So add it all together they would equal – I
think they would equal ninety-si –
[Brandon writes 16 into each sixth of the paper.]
Ball: Mm-hmm.
Brandon: No. Ninety-six?
Ball: Mm-hmm.
Brandon: So in – if I put seventeen then they’ll be one-hundred and something,
so I’ll be over.
Ball: Okay. So let’s just keep these available for a minute. So you have this
one – here’s the one you made in halves,
Brandon: Mm-hmm.
Ball: okay, here’s the one you made in fourths, and this one has what? Thirds
and sixths?
Brandon: Mm-hmm.
Ball: Okay, so I’m going to write a fraction down and then I’d like to see if you
can show me what part of one of these pieces of paper you could use for that
fraction.
Brandon: Mm-hmm.
Ball: So give me a clean piece of paper please. I’ll show you – we’ll start with
one that’s easy.
Ball: So here’s one you already did, just you see what I’m saying. If I write
that fraction, can you – here, let’s put these on your side. Can you pick up a
piece of paper and show me which – how to use one of those pieces of paper
to show one-half?
[Ball writes $\frac{1}{2}$ on the paper.]
Brandon: This one?
Ball: Okay. So now I’m going to write a different one, okay? Can you use one of your pieces of paper to show that?

[Ball writes \( \frac{2}{4} \) on the paper.]

Brandon: I think this one solves both of those.

Ball: Okay, how?

Brandon: Because when one-half is two-fourths – when it’s reduced – so it’s the same thing, only just the numbers are bigger.

Ball: Okay. What about this?

[Ball writes \( \frac{2}{3} \).]

Brandon: I think [when it] – this in th – in thirds . . .

Ball: Mm-hmm

Brandon: . . . this in thirds, so it would be these two – these two parts would be two-thirds

Ball: Okay. What about . . . ?

[Ball writes \( \frac{4}{4} \).]

Brandon: That would equal a whole.

Ball: Okay. Okay so how do you know that was a whole?

Brandon: Because, umm, all fractions are pieces of a whole, so – just – I knew that half wouldn’t be, so – but all of those just equal into one.

Ball: Okay. So what if I wrote down this fraction? Could you show me that with the paper?

[Ball writes \( \frac{3}{2} \).]

Brandon: That would equal a whole.

Ball: Okay. Okay so how do you know that was a whole?

Brandon: Because, umm, all fractions are pieces of a whole, so – just – I knew that half wouldn’t be, so – but all of those just equal into one.

Ball: Okay. So what if I wrote down this fraction? Could you show me that with the paper?

[Ball writes \( \frac{3}{2} \).]

Brandon: Wait, how would it go into it?

Ball: Could you show me that much paper with the . . . ’Cause you’ve been showing me one-half of a piece of paper, you showed me two-thirds of pa – of a piece of paper. Can you show me that much paper?

Brandon: Mm-hmm.

Ball: Can you put your paper up here?

Brandon: Mm-hmm.

Brandon: You would have to put two into three in order to find what – how mu – how mu – how much a piece of pach – paper would be. So it’d be one and a half.
15. INTERVIEW WITH BRANDON

[Brandon has written out the long division of 3 by 2:

\[ \begin{array}{c|cc}
& 1 & \\
\hline
2 & 3 \\
\hline
2 & 1
\end{array} \]

Ball: Okay. So how would you show that with the paper?

Brandon: It’s the whole and this part would be a half.

Ball: Okay. How would you read that number that I wrote? Can you read that number?

Brandon: Three-twos, I think.

Ball: Okay. So, umm, let’s, uh, work on the board for a few minutes. It’s getting kind of boring sitting here. Umm, where’s our chalk? I thought we had some chalk somewhere. Here. So, umm, here’s where we have a bunch of fractions we were just talking about. Can you, umm, take, let’s say one-half and two thirds. Could you write them up there?

[Brandon writes \( \frac{1}{2} \) and \( \frac{2}{3} \) on the chalkboard.]

Ball: Okay. So now which one of those two do you think is great — is larger?

Brandon: Larger? Umm. I think one-half — I think two-thirds?

Ball: Why do you think two-thirds is larger?

Brandon: Because half would just be half of something, but one-and-a-half is — is half a third, so — but it’s two — so, two would be bigger.

Ball: Okay. What about if we add, uh, three-fourths to that? Where would you put three-fourths?

Brandon: Let me see.

[Brandon writes \( \frac{3}{4} \) to the right of \( \frac{1}{2} \) and \( \frac{2}{3} \).]

Ball: Okay. So what are you saying about three-fourths?

Brandon: I think this would be a little bit bigger.

Ball: Than?

Brandon: Than... Three-fourths would be bigger than two-thirds ’cause, umm, it’s just — this is just over one-and-a-half by a half, but this is over — this — ’cause this is two thirds. Two-thir — I mean two fourths would be half, but three is over two so it’s over half by one...

Time: 00:27:20

Ball: Okay. All right, let’s sit back down again for a second. I’m going to show you some pictures of some different things. We’ll get away from the paper
folding for a minute. And then maybe we’ll do a little more comparing, like we were – remember we were doing it before this morning.

Brandon: Mm-hmm.

Ball: So, umm, like this for example. Can you tell me if this whole thing is the whole?

[Ball shows Brandon this figure:]

Brandon: No.

Ball: Okay. So can you tell me what that fraction would be?

Brandon: It would be . . .

Ball: The shaded part is what I’m talking about.

Brandon: Oh.

Ball: Sorry.

Brandon: Sixty percent?

Ball: Fifty percent?

Brandon: Sixty . . .

Ball: . . . Percent?

Brandon: Mm-hmm.

Ball: Why do you think it’s sixty percent?

Brandon: ’Cause five – it’s five whole things and all those equal to one, so each one is worth twenty percent.

Ball: Mm-hmm.

Brandon: So since it’s three of those it’s sixty.

Ball: Okay. Is there a fraction you could write for that?

Brandon: Mm-hmm.

[Ball writes $\frac{60}{100}$]

Ball: Okay. What if somebody wanted to write a fraction that had a five in the denominator since, like you said, there’s five parts? What fraction could you write if that was a five in the denominator?

[Ball writes $\frac{3}{5}$]

Ball: Okay. What if there was a ten in the denominator? Could you make a fraction that had a ten in the denominator?
Brandon: Ten? Would you – would you call these ten?

Ball: No, these would – You have five parts. How could you make it so that, umm, you represent it as a number of tenths? You can change the drawing if you want to.

Brandon: I think you can dr – you draw a line through this.

Ball: Okay, go ahead. You can do that.

Brandon draws a line through the figure:

Ball: Okay. So now could you write a fraction that was over ten? To –

Brandon: Yeah.

Ball: – represent the shaded part?

Brandon: Mm-hmm.

Brandon writes $\frac{6}{10}$.

Ball: Okay. How did you decide how to do that?

Brandon: 'Cause if – when I put a line through it, it makes it ten so this, even though these – all this is – is six, it’s six of them shaded with the line going through it. And without the line it’s – it’s just, umm, three over five.

Ball: Okay. Umm, of these three fractions you’ve written, which one’s the largest?

Brandon: Three-fifths?

Ball: Why is it the largest?

Brandon: 'Cause three-fifths is like – like we said earlier, it – 'cause one-hundredths are really small...

Ball: Mm-hmm

Brandon: I mean it – In – I – my opinion it’s not – it’s not about the numerator, I think it’s about the denominator.

Ball: Tell me – I know you were telling me that earlier, so keep going ’cause we didn’t really get to talk about that.

Brandon: Okay. So this would –

Brandon: That would be fifths
Brandon: and – this is tenths,

Brandon: so – so it’s six of that, even though the numer – the numerator’s bigger than this numerator, it – my opinion is that the denominator – how big the denominator determines how – how big the fraction – the whole fraction is.

Ball: So, what were we talking about earlier? Do you – is that paper around still? We were comparing, umm, let’s pursue this a little bit. Is that one-half and seven-eighths, or something like that? Or one-quarter and seven-eighths?

Brandon: Oh. They were on the – they were on those little cards.

Ball: Oh yeah, that’s right. Is it over by you?

Brandon: I’m not sure.

Ball and Brandon transition from working on the projector to working on the chalkboard. In this next segment Ball continues to lead Brandon through a series of activities where he draws pictorial representations of selected fractions to explain his understanding of the numerator and the denominator.

Ball: Here they are. Yeah. Here, why don’t – You want to put them up on the board?…Okay. So I asked you, I think, to put them up and put the smaller one on the, uh, right. I meant on the left, yeah. So you put them up, like… oh okay, is that what you meant before?

Brandon: No.

Ball: Okay.

Brandon: ’Cause this would be…

[Brandon places the 7/8 card on the left of the 1/4 card on the board.]
Ball: Okay, you put them sm – okay, right, sorry. Okay. So then tell me again what you were explaining, because I didn’t f – I didn’t completely – we didn’t get to really finish talking about it.

Brandon: Umm. What I understand is that fourths – fourths would be like that

[Brandon draws:]

Brandon: and eights is like this.

[Brandon draws:]

Ball: Okay.

Brandon: So s – Like, I said, umm, I think that f – the d – the, umm, denominator determines how big the whole fraction is.

Ball: Okay. So where does that lead you with these two fractions?

Brandon: 'Cause fourths – 'cause, like, fourths are bigger than eights so –

[Brandon colors the circle:]

Brandon: so even though it’s just…

[Brandon colors the circle:]

Brandon: So I think that –

Ball: So that’s your seven-eighths…

Brandon: Uh-huh. And this is one-fourth.
Ball: Okay. But when I look at your drawings, it looks like you’ve shaded more for your picture of seven-eighths than for one-fourth. That’s the part I’m not completely understanding.

Brandon: ’Cause – because it’s – you have fourths – I meant – I mean eighths is – eighths are a lot smaller, so seven of them would have to – you have to shade in ’cause you couldn’t put seven into four.

Ball: Okay. So then you have this piece of paper from earlier you – remember where you divided the paper into fourths and you told me that one of these could also be – you could write it as two-eighths, right?

Brandon: Mm-hmm

Ball: So what if we say instead of one-fourth, I take – ’cause you said that would be the same, right?

Brandon: Mm-hmm

Ball: Do you s – do you still think that’s the same?

Brandon: Right.

Ball: So what if I put two-eighths here instead?

[Ball moves the $\frac{1}{4}$ card down and writes $\frac{2}{8}$ to the right of the $\frac{7}{4}$ card.]

Ball: Now how would you compare seven-eighths and two-eighths? ’Cause then the denominators aren’t different?

Brandon: I think that seven-eighths would be bigger.

Ball: Okay.

Brandon: Because they’re – they both have eighth, but, umm, si – I mean seven would be bigger than two. They have the same denominator.

Ball: Okay. So in general, when they have the same denominator, how do you compare them?

Brandon: By the numerator.

Ball: But when the no – denominators are different, what is it you do?

Brandon: I compare them by the denominator.
Ball: Okay. So then this is the part I’m not — I don’t think get completely, ’cause if I understood you correctly, you said two eights was the same as one-fourth. Or were you not saying that?

Brandon: Umm. It’s —

Ball: Do you have a picture of two-eighths up on the board? Can you make a picture of this? [She points to the $\frac{2}{8}$ written on the board.]

Brandon: Okay.

[Brandon draws on chalkboard:]

Ball: Okay.

Brandon: So . . .

[Brandon finishes his drawing.]

Ball: Okay. So now this is your picture of one-fourth. Can you label it?

Brandon: Mm-hmm.

Ball: Your picture – this is a picture of . . . Okay. So now, what I’m trying to understand is when you compare these two [pointing to Brandon’s drawings of $\frac{1}{4}$ and $\frac{2}{8}$], what do you conclude about which one’s bigger?

Brandon: How do I determine it?

Ball: Yeah. How do you decide? Like, which one is bigger? ’Cause I thought you were saying they were the same ’cause you labeled this one one-quarter and then you labeled this one two-eighths, so I thought you were saying those are different ways to write the same amount? [Showing Brandon the paper he had written on previously showing that $\frac{1}{4}$ is the same as 0.25]

Brandon: No, it’s ’cause this would be bigger, but it can be — what I’m saying is that it can go smaller.

Ball: Say more about – what do you mean, “it can go smaller”?

Brandon: Like . . . I can’t really decide on how big they are when they’re reduced.

Ball: Mm-hmm. When you look at these two pictures, which one do you think is greater: two-eighths or one fourth?

Brandon: Umm . . . One-fourth?

Ball: Why do you think one-fourth?
Brandon: Umm. ’Cause it has – it has bigger chunks into it to make fourths, so – but these are all, like li – sorta small, so just one out of four is bigger than two out of eight.

Ball: Okay. Let’s, umm, let’s go over and use this line. Remember when I talked to you on the phone and you said you don’t usually use the line so much?

Brandon: Oh a number line?

Ball: Yeah. Or do you – did you say you did use it?

Brandon: I don’t think we used it.

Ball: Okay. I’m just going to mark a few points and then we can go from there. So I’m going to call that point zero, and then I’m going to call this one one, okay? And then I’ll call this one two, and that’s all I’m going to put on there for right now, okay?

[Ball draws on the board:]

Brandon: Mm-hmm.

Ball: So. Like where would three be if we were to put it on?

[Brandon adds a 3 to the number line:]

Ball: Okay. Now do you think there’s some numbers between these?

Brandon: Mm-hmm

Ball: Like what?

Brandon: Umm . . .

[Brandon begins to draw on chalkboard. He adds three lines between 0 and 1.]
15. INTERVIEW WITH BRANDON

the middle, between 0 and 1]? If this is one – this still has to be one and that’s zero so what would this be? [Again, referring to the middle line.]

Brandon: I think this would be half.

[Brandon writes:]

\[
\begin{array}{c}
0 \quad \frac{1}{2} \quad 1 \\
\end{array}
\]

Ball: Okay. So then . . .

Brandon: Just half.

Ball: Just half. And why did you think that would be one-half?

Brandon: Because this would be, umm, this would be . . .

[Brandon writes:]

\[
\begin{array}{c}
0 \quad \frac{1}{2} \quad \frac{3}{4} \quad 1 \\
\end{array}
\]

Brandon: So it’s – so like this is half [pointing to the middle line]. . .

Ball: Mm-hmm

Brandon: This is like – this – this side is bigger – I mean the same – they have the same side, but this is closer to one, so . . .

Ball: Okay. Then what’s that one there then?

[Brandon writes:]

\[
\begin{array}{c}
0 \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{4} \quad 1 \\
\end{array}
\]

Brandon: One-fourth.

Ball: Okay. And how did you decide to label this one one-fourth and this one three fourths?

Brandon: 'Cause one fou – I mean . . . 'Cause four of these equal one, so this is just one section of it, so it’d be one, and this would be half point, and this is fourths, and then this would equal one.

Ball: What did you mean four of them equal one? What did you mean by four of them?

Brandon: Four is . . . So – I probably should change it. I need an eraser.

Time: 00:38:34

[Brandon changes his number line:]
Ball: Why did you change that?
Brandon: 'Cause, umm, I wanted them all to be fourths...
Ball: Oh.
Brandon: ...so we wouldn’t get confused, so...
Ball: How did you decide the denominator should be fourths? Why not fifths or sevenths or something like that?
Brandon: Well I think... Because in some numbers you can’t – you can’t put the numerator into the denominator...
Ball: Mm-hmm
Brandon: ...so – but this would be easier.
Ball: So, was one-half wrong where you had it? I mean is – is this point – could you call this one-half also?
Brandon: Mm-hmm
Ball: So why don’t you put it back, like underneath it?
[Brandon had erased the $\frac{1}{2}$ he had written and replaced it with $\frac{2}{4}$. Ball was now asking that he add $\frac{1}{2}$ back to his number line, positioning it just below the $\frac{2}{4}$.]
Brandon: Okay.
[Brandon writes:]

\[0 \quad \frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad 1 \]
\[\frac{1}{2}\]

Ball: Is there any other fraction you could call that point besides two-fourths and one-half?
Brandon: Umm, yeah. I call it – you can call it fifty-out-of-a-hundred, or...
Ball: So why don’t you write fifty-out-of-a-hundred too, then.
[Brandon writes:]

\[0 \quad \frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad 1 \]
\[\frac{1}{2} \quad \frac{50}{100}\]

Ball: So this is – goes back to the thing you were telling me earlier in general. Like you can take some number and then half of it and then...
Brandon: Mm-hmm
Ball: Okay. So, uh, what number would be here? [She draws a mark on the number line, half way between the 1 and the 2.] Here, I’ll use a different color from you. What number would you say would be here?

Brandon: This would be half point ‘cause it’s the same size or bigger, so, umm…

[Brandon writes:]

\[1 \quad \frac{1}{2} \quad 2\]

Brandon: So it’s like – it’s not exactly right here [pointing to a space between \(\frac{1}{2}\) and 2] where it’s like three out of four and then it’s not exactly one out of four [pointing to a space between 1 and \(\frac{1}{2}\)], so it’s half.

Ball: Are you saying it’s the same number as that number? [She points to the \(\frac{1}{2}\) mark between the 1 and the 2 and refers back to the \(\frac{1}{2}\) mark between the 0 and 1.]

Brandon: Yeah.

Ball: ‘Cause you have one-half there.

Brandon: Mm-hmm.

Ball: But this number is greater than one. See, it’s past the one [indicating that the new mark between 1 and 2 lies to the right of the 1 mark].

Brandon: Yeah.

Ball: So, is it just one-half?

Brandon: It’ll be one-and-a-half.

[Brandon writes:]

\[1 \quad \frac{1}{2} \quad 2\]

Ball: Oh, it’d be one-and-a-half. Okay, so if we repeat the same thing you did before, could you label that one and that one?

Brandon: Mm-hmm.

Ball: What would that be?

[Brandon writes:]

\[1 \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{4} \quad 2\]

Time: 00:40:42

Ball: Okay. So, umm, is there anything else you could write – you know how you wrote different ways to write one-half? Are there for one-and-a-half?
Brandon: Yeah.

[Brandon writes:]

\[
\begin{array}{cccc}
1 & \frac{3}{4} & \frac{1}{2} & \frac{3}{4} & 2 \\
1 & \frac{17}{24} & \frac{31}{58} & \frac{31}{68} \\
\end{array}
\]

Ball: Do you need the one in front of it, or you don’t need the one in front of it?

[Brandon shakes his head in the negative.]

Ball: So this is just seventeen thirty-fourths?

Brandon: Mm-hmm. It’s – that’s half.

Ball: Okay.

Brandon: Mm-hmm.

Ball: But it’s greater than one, right? So . . .

Brandon: Mm-hmm.

Ball: So . . .

Brandon: It’s one.

[Brandon writes:]

Ball: Okay. All right. So, umm, can you think of any other fractions that you could put up here? Like, is there any fraction that goes between one-fourth and two-fourths, for example?

Brandon: Umm, mm-mm. No.

Ball: No? Er, have we put all the fractions up here that we can?

Brandon: Yeah.

Ball: Well what if we wanted to put one-third up there? You think it couldn’t – we couldn’t put it on there?

Brandon: Sure, but we would probably have to put them into thirds instead of fourths.

Ball: Okay, so you can do that. Here, let’s – what color were you using? Pink? Here, just use this.
Brandon: Okay.
Ball: Can you – can you mark it so that you’d be able to put thirds one there?
Brandon: Do I need to erase this?
Ball: You don’t have to erase it.
Brandon: Okay.
Ball: You can kind of ignore what you’ve done; now concentrate on this. Okay, so what would you label those two points?

Brandon writes:

Brandon: This section would be one-third. This would be two-thirds. And then this would be three-thirds right here.
Ball: Okay.
Brandon: Whole.
Ball: If you wrote this with – in halves, what number would you write there? So you wrote three-thirds, could you write some number of halves there? Something over two?
Brandon: Mm-mm
Ball: Could you write something over four there?
Brandon: Over four? Yeah.
Ball: What would you write over four?
Brandon: For half?
Ball: …yeah. For one. Like instead of three thirds, could you write a fraction that’s . . .
Brandon: That’s . . .
Ball: …over four, like that?

Ball adds $\frac{6}{6}$ to the number line:
Brandon: No, 'cause this is all one, so three-thirds or four-fourths, or . . .

Ball: Okay. You can write four-fourths.

Brandon: Or . . .

[Brandon writes:]

Brandon: Eight-ninths.

Ball: Okay. That’s supposed to go, like here? It’s the same?

Brandon: Mm-hmm.

Ball: Okay. So now I’m going to give you some cards with fractions on them, and you don’t have to find exactly where they would go, but I’d like you to – here I’ll show you. Where’re those ones with the magnets that we had? These? Okay. So here I just want you to put it sort of in the right place where you think it would go. Like is it more than one, is it less than one? Is it more than a half, is it less than a half? It’ll be hard to tell exactly where it should go, but about where do you think that fraction goes? What is it? Can you read it to me?

Brandon: Eight-ninths.

Ball: Okay. So wh – approximately where do you think you would put that? For example, is it – which side of one does it go on?

Brandon: It goes right here because if it’s – it’s not – it’s not – it’s – it’s not nine-ninths . . .

[Brandon places the \( \frac{8}{9} \) card directly to the left of the 1 on the number line.]

Ball: Okay.

Brandon: . . .so it wouldn’t be one.

Ball: So nine-ninths would be one? And how do know it’s more than a half? ’Cause you put it – like here’s a half. You put it quite far over from a half.

Brandon: Yeah. ’Cause four-and-a-half is half of nine, so – but this is eight so it’s over –

Ball: Okay.

Brandon: it’s over . . .

Ball: All right. Where would you put this one? Can you read that?

Brandon: Three-fifths.
Ball: Yeah. Where would you put that one?
Brandon: [right] two, three, four . . . Hold on.
Ball: I mean again roughly. Like is it close to one? Is it close to zero? Is close to a half?
Brandon: So these lines right here would be representative – representative –
Ball: So about where do you think it goes? Which side of one-half does it go on?
Brandon: I think it’s a li – it’s a little over half.
[Brandon places the $\frac{3}{5}$ card directly to the right of the $\frac{1}{2}$ on the number line.]
Ball: Okay.
Brandon: ’Cause two-and-a-ha – two-and-a-half is five – is half of five, so . . .
Ball: Okay.
Brandon: . . .but it’s three so it – it would be over.
Ball: Where would you put that one?
Brandon: Right here.
[Brandon places the $\frac{19}{19}$ card directly under the 1 on the number line.]
Ball: What is that?
Brandon: Nin – nineteen-nineteenths.
Ball: Okay. Why does that go there?
Brandon: ’Cause it’s whole.
Ball: Okay. And where would you put this one? What is that fraction?
Brandon: Three-twos?
Ball: Okay. Where would you put it?
Brandon: So, okay. So which one is the whole number? Which . . .
Ball: It’s divided into parts that are halves. So . . .
Brandon: To find this answer – to find this answer you would have to divide two into three. So . . .
[Brandon writes on chalkboard:]
\[
\begin{array}{c}
\frac{1}{2} \\
\frac{3}{2}
\end{array}
\]
Brandon: So it would be one-and-a-half.
Ball: Okay. So where would you put it?
Brandon: [Gap in audio] – put this right here.

Ball: Okay. I have a – one or two more to show you.

[Ball goes for more cards.]

Ball: Oh. Where would you put that one. What is that?

Brandon: Zero-fourths.

Ball: Yeah. Where would you put that?

Brandon: I’d put this right here.

[B Brandon places the $\frac{0}{4}$ card directly under the 0 on the number line.]

Ball: Why?

Brandon: Because, umm, it’s not a percentage yet, so its –

Ball: It’s not a what?

Brandon: I mean it’s not a part of it.

Ball: Uh-huh.

Brandon: It’s not like – the numerator doesn’t have a number on top of it.

Ball: And what does that tell you?

Brandon: It tells me that it’s not part of a fraction yet, so it would just be – it would just be zero.

Ball: Okay. Do you want to a really strange one, and then we can stop with this? What does that say?

Brandon: Two – eight – two-hundred-eighteenths over two-hundred-six?

Ball: Now I just want to know roughly, do you think it’s less than one? More than one? Is it more than two?

Time: 00:47:05

Brandon: I think it’s more than one 'cause it’s – 'cause two-hundred-and-six can’t go into two-hundred-eighteen two times, so . . .

Ball: Okay. So where would you put it, about?

Brandon: I would put it – just randomly I would put it right here.

[Brandon places the $\frac{218}{206}$ card directly to the right of the $\frac{11}{2}$ on the number line.]

Ball: Why did you put it so close to two? Like how did you decide to put it all the way over there?

Brandon: ’Cause I’m not really sure ’cause it’s not really – it’s not – I’m not sure if it’s over half or before half.

Ball: So what if I asked you to write a fraction, let’s say, that was – well let me write it in a different spot. What if here where you’ve been writing these
other fractions I asked you to write a fraction that was the same as one that had two-hundred-six in the denominator? What would you put in the numerator if you wanted it to be the same amount as all these other numbers here?

[Brandon writes \( \frac{206}{206} \)]

Ball: Why did you write that?

Brandon: If I wanted it to be whole, then it would have to be two-hundred-and-six.

Ball: Okay. So now if you look at this one, does that help you know about how big that one is, or not really?

Brandon: Not really.

Ball: Okay. Why don’t – You want to switch and do something else for a while?

Brandon: Sure.

Ball: Did you say you hadn’t been working with number lines or you had worked with number lines?

Brandon: We haven’t.

Ball: How – was that hard to do, what we were doing?

Brandon: No, ’cause – ’cause most of – in my opinion it’s sort of like common sense.

Ball: Uh-huh.

Brandon: It’s like – if it’s one then it would just be – if you put, like, four points – it depends on how much points you put.

Ball: Mm-hmm.

Brandon: If you put five then we’ll know that it’ll take a certain amount – five to equal one, or four – you put four points then it’ll be –

Ball: Well, you know this thing we were having over here about the eighths?

Brandon: Mm-hmm.

Ball: I wonder how that looks on here. So you said, umm, you were trying to talk about two-eighths and one-fourth. So where would you put two-eighths on this number line?

[Brandon divides the sections between 0 and 1 on the number line into eighths.]

Ball: Okay, so . . .

Brandon: The white points would be – these are eighths.

Ball: Okay. So can you label them?
Brandon: Mm-hmm

Ball: I think we're getting too many things on here, maybe.

Brandon: Probably.

Ball: It's hard to keep track of. Umm, maybe let's just make a clean one 'cause I think we have too many things on here and I think y – it was hard to see what you were doing. Let's come back – well let's go over here and we'll just work on a little piece of it and see if we can talk about this eighths. So… There's zero, and I'll just do it between zero and one this time, okay?

Brandon: Mm-hmm.

Ball: So I'm just going to put back a couple things we had. We had one half, and we had th – uh, what was this?

Brandon: Umm, three-fourths

Ball: Three-fourths. We also had two-fourths. And we had one-fourth. That's all I'm going to put on for right now, okay? Or maybe we'll put a couple of your whole ones.

Brandon: Mm-hmm.

Ball: What did you have? Four-fourths?

Brandon: Yeah.

Ball: Okay. So is that enough now just to get oriented?

Brandon: Mm-hmm.

Ball: So now try to make the eighths. Okay, do you want a colored one? So can you make that so you can represent eighths? Just do it carefully 'cause you were right, you wanted to make eight parts but I think you lost track a little bit.

Brandon: Two…

Ball: So now try to make the eighths. Okay, do you want a colored one? So can you make that so you can represent eighths? Just do it carefully 'cause you were right, you wanted to make eight parts but I think you lost track a little bit.
Ball: Okay. So, umm, this point right here [pointing to the mark] is what?
Brandon: It’s half.
Ball: It’s half?
Brandon: Mm-hmm.
Ball: And what’s the pink one you just drew? What’s that?
Brandon: That’s half point.
Ball: That’s a half point? So in eighths what would you write?
Brandon: Four-eighths.
Ball: Okay. So what about this one right here?
Brandon: This is two-eighths.

[Brandon has written:]

Ball: Okay. So now you’ve got two-eighths and one-fourth at the same point, right?
Brandon: Yeah.
Time: 00:51:58
Ball: So how does that go back with what we were talking about with the pictures? 'Cause here, I think you were telling me that one-fourth was more, 'cause the fourths were bigger chunks.
Brandon: Mm-hmm.
Ball: But here you’ve got them at the same point, so I’m curious about that.
Brandon: Because – 'cause you put, like – if – it depends, like – where – how – how big are the points, so if they were in fourths, then one-fourth would be bigger . . .
Ball: Mm-hmm.
Brandon: . . .because eighths are like – fourths – some fourths are like this, but then eighths are like this.
Ball: But didn’t you –
Brandon: So – but the space between them would be bigger.
Ball: Right. But you have that, right? See here you have one-eighth and then two-eighths. You have two-eighths right here at the same place. Is that in the wrong place, that two eighths?
Brandon: No. One-fourth is – one-f... Because I put them – this is in eighths, but this – if you put them in fourths then it would be right here.

Ball: Okay. So are they the same or they’re not the same?

Brandon: They’re not the same.

Ball: And why, on this drawing, do they come out looking like they’re the same? On this number line?

Brandon: ’Cause if you s – if you put them in fourths then this would be one . . .

Ball: Mm-hmm.

Brandon: [counting the fourths along the number line]...two, three, and then . . .

Ball: What you have, right? You have one-fourth, two-fourths, three-fourths.

Brandon: All right, and then one whole . . .

Ball: Right. So here you have – when you divide it into eighths it looks like you’ve got one-eighth, two-eighths – is this right? – three-eighths, and the you already wrote four-eighths –

Brandon: Four . . .

Ball: – then you would have five-eighths . . .

Brandon: Six . . .

Ball: ...then six-eighths, and then seven eighths? Is that – is that correct?

Brandon: Mm-hmm.

[Ball has written:]

Ball: But then you do have – you have four-eighths at the same place as one – well what’s bigger: one-half or four-eighths?

Brandon: I mean, the numbers are bigger, but they’re both the same cause they’re both the same ’cause they’re both half.

Ball: Okay. So those you see as the same. Is two-fourths the same also?

Brandon: Mm-hmm.

Ball: Now back to this. You think these are different though –

Brandon: Yes –

Ball: – is that right?

Brandon: – yes because – I put – these are in fourths . . .
Brandon: ... what determines how big the fraction is that if the space is.

Ball: Mm-hmm.

Brandon: 'Cause from here [pointing to zero and referring to the distance between 0 and $\frac{1}{4}$] it's like -- this is like one-fourth and two-fourths and three-fourths and one whole [counting up the number line by fourths], so the space -- the space between these are eighth [pointing to the distance between 0 and $\frac{1}{8}$], so -- but this is one-fourth so they're in -- since it's in eighths, the spaces in between it -- it is smaller, so that's why one-fourth would be bigger [i.e. because the space between 0 and $\frac{1}{4}$ is bigger than the spaces between the eighths, one-fourth is bigger than two-eighths].

Ball: Okay. Let's -- let's -- why don't we leave our number lines and drawings on the board and we'll switch gears a little? We've probably done enough of that for a while.

Time: 00:54:40

Brandon and Ball: walk back to the projector.

Ball: Okay. Did we get s -- do we have any clean paper left?

Brandon: Yes.

Ball: Okay. Now I'm just going to show you some pictures that -- some of them are easier and some of them are harder...

Brandon: Mm-hmm.

Ball: ... and I'd like you to try to think about how much of the whole it is and what fraction you could use to express that, okay? So, how 'bout this one?

[Ball shows Brandon a figure:]

Ball: Let's put a piece of paper under it in case you want to write something. So the triangle -- the big triangle is the whole and I'm interested in what you think about the shaded part.

Brandon: It's half. [He writes $\frac{1}{2}$ under the picture.]

Ball: Okay. And how did you decide that?

Brandon: 'Cause it's bo -- it's evenly split --
Ball: Mm-hmm.

Brandon: – so one side is the same as this side, so...

Ball: Okay. What about this one? So the whole is this whole rectangle –

[Ball shows Brandon a figure:]

Brandon: Mm-hmm.

Ball: – okay?

Brandon: So would this be in – what’s – is it – would this be in fourths, or...? I’m not sure.

Ball: What are you not sure about? What is your question?

Brandon: Because these don’t have a line in between the middle, but these two do, so...

Ball: You can do – you – you need to do something to the picture, you can.

Brandon: Okay.

Ball: If that helps you decide what you want to call that.

[Brandon draws a line through the rectangle:]

Ball: Okay. What did you do?

Brandon: I put a line though it.

Ball: And why – why did you want to do that?

Brandon: ’Cause this part doesn’t have a line through it so it would – so – but this part does, so it wouldn’t make sense for just this part to have a line through it and not the rest.

Ball: Okay. So now would you be able to decide how much the shaded part is?

Brandon: Mm-hmm.

Ball: What?

Brandon: It’s one – it’s one-sixth.

Ball: Can you write that?
Ball: Okay. Okay, this one – this one might be a little bit trickier. Maybe not, I don’t know.

[Brandon writes $\frac{1}{6}$.]

Ball: Okay. Okay, this one – this one might be a little bit trickier. Maybe not, I don’t know.

[Ball shows Brandon a figure:]

Ball: So the whole is the whole rectangle, okay? And what you want to try to figure out is how much is shaded.

Brandon: Mm-hmm.

Ball: You can talk about it as you think about it, if you want to.

Brandon: I – I’m not really sure ’cause – I mean t – this – if this was a fraction [pointing to the second square of the picture], this [pointing to the last square of the picture] would be half of it ’cause it’s sh – and this would be half, so . . . But this isn’t shaded, so I guess this is one and –

Ball: Why don’t you keep a little record of what you’re saying? So you’re saying this is one . . .

[Brandon writes:]

Brandon: Yeah. And that would be the half – that would be half.

Ball: Mm-hmm.

Brandon: And this would be half, so . . .

Ball: Okay. But you’re – what are not sure about then?

Brandon: ’Cause – ’cause this isn’t shaded, so – but this i – this part is shaded.

Ball: Okay. So if you just looked at these two squares right here?

Brandon: Mm-hmm.

Ball: How much of the whole are these two? Forget about the shading. So you have this whole rectangle. What if – how much of the whole are these two together?

Brandon: It’s two squares.
621 Ball: Mm-hmm.
622 Brandon: Okay.
623 Ball: A – and how much of the whole is that? What fraction of the whole are those two squares?
624 Brandon: Which fraction is shaded?
625 Ball: Which-uh, yeah. Like if we shaded this . . . And there’s your original rectangle. What fraction of the whole would be shaded?

[Brandon draws:]

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626 Brandon: Two-fourths.

Time: 00:58:27
627 Ball: Two-fourths, okay. But you just said only half of that is shaded, right? So what’s half of two-fourths?
628 Brandon: Two – I mean half is two-fourths. One-fourth?
629 Ball: Okay. So here you’ve got one-fourth is shaded and one-fourth isn’t?
630 Brandon: No.
631 Ball: No?
632 Brandon: I’m not really s – I’m not sure.
633 Ball: Okay. Why don’t – should we leave that one? What about this one?

[Ball shows Brandon a figure:]

634 Brandon: I’m guessing they’re six. They’re six ’cause –
Ball: Do you want to draw some lines again?
Brandon: Sure.

[Brandon draws:]

Ball: Okay. So now would be able to say how much of the – this whole is shaded?
Brandon: This part is, like – so all of these are eighths, so I think – so five eighths?
Ball: Okay. Does it matter that they’re not next to each other?
Brandon: Mm-mm
Ball: How would you write five-eighths?
Brandon writes $\frac{5}{8}$.
Ball: Okay [puts a new example on the projector].

Brandon: Those would be – that would be half ’cause there’re four segments of it –
Ball: There’re what?
Brandon: There’re four pieces of it and they all come together to be one.
Ball: Can you draw that? Can you – you want to put any lines on to show me what you mean? Or you’re saying one, two, three, four [pointing to the different areas on the picture]
Brandon: Yeah. It – I mean they’re different shapes, but there are four segments of it and they come together to make one square, so –
Ball: Uh-huh.
Brandon: – but two of the segments are shaded, so it would be two-fourths, or half.
[Brandon writes $\frac{2}{4}$ under the figure.]
Ball: Okay, how did you decide that? Because…
Brandon: Because since it’s four of – segments then – and two are shaded so two out of four would be half.
Ball: Earlier when you were folding the paper you made the big point to me that, umm, like a little kid might fold them and not mi – might not make the parts the same size.

Brandon: Mm-hmm.

Ball: Do you think these are the same size there?

Brandon: Mm-mm.

Ball: You don’t think they’re the same size. Which ones do you think are different?

Brandon: These two – I mean these two from this – from there [pointing to the two small rectangles that comprise the shaded square] or these two from this – from this . . . [pointing to the unshaded quadrilaterals along the bottom and right side].

Ball: Okay. So how come you’re – how come you’re going to call those each one-fourth then if they’re not the same part – the same size?

Brandon: ’Cause one – they’re all one piece but there a line that puts them into – that separates them ’cause, it’s like . . . This is one,

[Brandon draws:]

Brandon: this is one,

[Brandon draws:]

Brandon: and then this part is . . .

[Brandon draws:]

Brandon: they’re all just kind of – there’s a line that separates them –

Ball: Mm-hmm

Brandon: – That puts them into fourths, but since they’re different sizes they’re four – they’re still four segments, so – and two of them are shaded, so it’s two-fourths.
Ball: Okay. Do we have more paper? Are we running out?
Brandon: Yeah.
Ball: We are running out?
Brandon: Mm-hmm. Well we still have a few.
Ball: Okay. We’re okay then. Okay. So if I write this fraction. Here, what is that?
   [Ball has written $\frac{3}{6}$.]
Brandon: Half. Three-sixths or half.
Ball: Okay. Well actually I was going to ask you about that. Earlier you said something about reducing fractions.
Brandon: Mm-hmm.
Ball: How would you reduce that fraction?
Brandon: Find a number that goes into both the numbers evenly.
Ball: Mm-hmm. So can you show me?
Brandon: So three – I picked three. Three can go into three one, and three go –
   [Brandon has written $\frac{1}{2}$ directly to the right of $\frac{3}{6}$.]
Ball: Okay. What if somebody didn’t believe you that one-half was the same amount as three sixths? Is there some way you could show them?
Brandon: Mm-hmm. Or you could just – if you multi – multiply these like –
   no, that’s wrong . . .
Ball: You can scratch it out if you want.
Brandon: Okay.
Ball: Okay. So how could you – what if somebody didn’t think those were the same? Are you saying that they’re the same?
Brandon: Mm-hmm.
Ball: How could – what if somebody didn’t believe you? ’Cause, I mean, a lot of people might say those numbers are a lot bigger.
Brandon: Umm. I think – I’m not sure. There’re so many ways, but . . .
Ball: Well what’s a good way, do you think?
Brandon: Umm. You could – I could draw – draw, umm, a picture to explain it.
Ball: Mm-hmm.
Brandon: Or something that it – that’ll show that three-sixths is the same as half.

Ball: So what do you think – what is one way that you could do it?

[Brandon draws:]

Ball: Okay.

Brandon: Those are sixths so . . .

[Brandon colors the rectangle:]

Ball: Okay.

Brandon: There’s the fraction for how much of it is shaded then it would be three-sixths

[Brandon writes under $\frac{3}{6}$ the rectangle.]

Brandon: and all together it’s six, so three would be half and the other three would be half.

Ball: Okay. All right. So why do you call that reducing?

Brandon: Reducing? Umm, ’cause you – if – ’cause some numbers can’t be reduced, or some fractions can’t be reduced, but redu – called reduce when it’s reducing when you make it smaller, but if it’s reduced then it’s already small.

Ball: Okay, but are you actually making the fractions smaller? Like is one-half smaller than three-sixths?

Brandon: I mean, they’re both the same, like, form, but they’re – they’re different numbers

Ball: Okay. S – can you write down a fraction that can’t be reduced? Can you think of one?

Brandon: Mm-hmm.

[Brandon writes $\frac{5}{6}$ on the overhead.]

Brandon: Five-sixths.

Ball: Okay, why can’t that be reduced?

Brandon: Because there’s no number than can into five and six evenly.
Ball: Mm-hmm. Umm, can you reduce fractions that are a little less familiar? Like, for example, if I wrote a number like...

[Ball writes $\frac{24}{42}$]

Ball: What does that fraction say?

Brandon: Twenty-four-s – er – uh – er – twenty-four-forty-twos?

Ball: Yeah. Can you reduce that?

Brandon: Yeah.

Ball: How would you do it? How could you tell so quickly that you could?

Brandon: Because they end and start with a number that – that – that a number can go into.

Ball: Okay. So what do you – what do you – what do you think you’d do to do it?

Brandon: One is that they’re both even.

Ball: Okay, and why does that tell you it can be reduced?

Brandon: Well, if they’re even, and some numbers that are odd can be reduced too, but...

Ball: Okay. But if they’re even you’re sure they can be reduced?

Brandon: Yeah.

Ball: Why is that?

Brandon: ’Cause if it ends with, like, a number with two, four, six –

Ball: Mm-hmm –

Brandon: – it can be reduced to... some could be even, but some numbers are odd that can be –

Ball: Okay.

[Brandon writes $\frac{6}{7}$]

Brandon: Six-sevenths.

Ball: Okay. How did you do that?

Brandon: ’Cause I picked a number that can go into t – twenty-four and forty-two, and it –

Ball: What number did you pick?

Brandon: Six.

Ball: You picked six?

Brandon: Wait no. Actually it’s...
Brandon: Four-sevenths.

Ball: Okay. What number did you pick?

Brandon: Six.

Ball: Okay. And what did you do?

Brandon: I... six can go into twenty-four four times, and six can go into seventy-two seven times.

Ball: Okay. All right. Umm, let's see. Are you tired? Do you want something to drink?

Brandon: Mm-mm.

Ball: No? Okay.

Ball: They're not being very good are they? Aren't they supposed to be quiet?

Brandon: They were quiet.

Ball: For a while. Maybe they'll be a little too quiet now. Okay. Well actually I was going to show something sort of tricky. I need another piece of paper. We're going to run out very fast. So just now you were reducing fractions...

Brandon: Mm-hmm.

Ball: ...but, umm, sometimes when I've been teaching kids, they do other things to reduce fractions and I wanted to show you something that I saw a student do, and I want to know if you think you could do it this way. So I showed him this fraction –

[Ball writes \( \frac{13}{43} \).

Ball: – And he said, “Well, I c – I know that that’s – I can reduce that just by crossing out, or canceling the numbers that are in the ones place”

[Ball crosses out the threes, changing the fraction to \( \frac{1}{4} \).

Brandon: Mm-hmm.

Ball: “... and so I can just do that and it will be one fourth.” Is that a correct way to reduce fractions?

Brandon: I can’t really say if it’s right or wrong cause I’ve never tried it.

Ball: Mm-hmm. What do – what would you try to – if you had to try to figure out, like, if that was a good method, or if it really was a method, what would you do to try to decide?

Brandon: Umm. I would see – I would see what number can go into those and see if it comes out to one fourth.
Ball: Mm-hmm. And does any number come to mind that you could do to divide into both of those to try to reduce it?

Brandon: No.

Ball: You can’t think of one, or there isn’t one, or what?

Ball: What’re you trying?

[Brandon has written $\frac{13}{43}$.

Brandon: I’m trying to see if thirteen can go into forty-three.

Ball: Can you think of a number that you can use to divide into thirteen that you could also divide into forty-three?

Brandon: Umm. Three?

Ball: Three would go into 13?

Brandon: No. I’m not – I don’t think – I’m not sure, ’cause – no, I don’t think any number can go into thirteen evenly.

Ball: Uh-huh. So then what does that tell you about this? [i.e. $13/43$]

Brandon: Umm. I’m not sure. I can’t find a number that’ll go into thirteen and thirty-three evenly – I mean forty-three evenly.

Ball: Okay. So what’s your view about this method right now? You’re saying you don’t know, right? Is that what you’re saying? Or are you thinking it doesn’t – it’s not a good idea?

Brandon: I’m thinking it’s not a good idea.

Ball: Uh-huh. Why a – why are you thinking that?

Brandon: ’Cause – I don’t know how, but I can’t – there – I don’t think there’s a number that can go into thirteen and forty-three evenly.

Ball: Okay. Have you done some adding and subtracting of fractions?

Brandon: Mm-hmm.

Ball: And you did – you showed me you were doing some dividing also, right?

Brandon: Mm-hmm.

Ball: What else? Have you multiplied fractions too?

Brandon: Mm-hmm.

Ball: You want to do a few of those now? Okay. Can I have a, umm – can you hand me one of those pens. And I think we’re okay with paper. Yeah. Thank you.

Ball: How would you do that problem? You can use your own. Here.

[Ball has written:]

$$\frac{2}{3} + \frac{2}{3} =$$
Brandon: It would equal four-thirds.

[Brandon writes his answer:]

\[
\frac{2}{3} + \frac{2}{3} = \frac{4}{3}
\]

Ball: How’d you do it?

Brandon: You added them, but some frac – some – along the line there’s going to be a problem that – where the denominators aren’t the same . . .

Ball: Mm-hmm.

Brandon: . . . so you would have to make the denominators the same.

Ball: Uh-huh. But if they’re the same, then what?

Brandon: Then you can add.

Ball: Okay. Well what if somebody said the answer to this was actually four-sixths, what would you say? Do you see how somebody might get that?

Brandon: Mm-hmm.

Ball: What would you say about that?

Brandon: It’s not correct.

Ball: Why?

Brandon: Because, umm. ’Cause you don’t – you don’t add the denominator.

Ball: Okay.

Brandon: I think you just add the numerator.

Ball: All right. So let – want to try one where the denominators are different? Can you show me how you do that? Are you going to write one? Okay.

[Brandon writes \(\frac{4}{6}\) and then \(\frac{2}{3}\) directly under it.]

Ball: So you’re doing four-sixths plus two-thirds?

Brandon: Mm-hmm.

Ball: Okay. Can we – let’s – You want to write it horizontally and then show me how you would do it, or is this how you would normally write it, vertically?

Brandon: This way.

[Brandon writes:]

\[
\frac{4}{6} + \frac{2}{3} = \frac{4}{3}
\]

Ball: Okay. So what would you do?
Brandon: I would find a number that can go into three and six evenly.

Ball: Mm-hmm.

Brandon: The smallest number that can go...

Ball: So like what?

Brandon: Three.

Ball: Okay.

Brandon: Wait, no. I’m getting confused... like six. ‘Cause si – times two and then... You – whatever you do, whatever you multiply to get – get to the number, you do to the numerator, so...

Ball: Okay.

Brandon: ...it’s times would be times two by the numerator.

[Brandon has written:]

\[
\frac{4}{6} \times 1 \quad \frac{4}{6} \\
+ \frac{2}{3} \times 2 \quad \frac{4}{6}
\]

Ball: Mm-hmm.

Brandon: Then you would add.

[Brandon writes:]

\[
\frac{4}{6} \times 1 \quad \frac{4}{6} \\
+ \frac{2}{3} \times 2 \quad \frac{4}{6}
\]

Brandon: But that can be – that can be put into another number.

Ball: Okay, what did you get for that?

Brandon: Eight-sixths.

Ball: Okay.

Brandon: But that’s a improper fraction. When – it’s when the – the numerator is bigger than the denominator. So what – I would divide six into eight.

[Brandon writes:]

\[
6 \div \frac{8}{6} = \frac{6}{2}
\]
Brandon: Two – So you would get one-and-two-sixths, or one-and-one-third.
Ball: Okay. And why do you divide it to find out how to write it another way?
Brandon: Umm.
Ball: You’ve done that actually a few times. You did that with the three-over-two a couple times, right?
Brandon: Mm-hmm.
Ball: Why are you dividing it?
Brandon: ’Cause, umm – ’cause there’s a – a different way that the numerator can be – it can be smaller than the denominator. It can – it’ll be back to a whole number and a fraction of the whole.
Ball: Okay. I’m going to write a addition problem, okay? How ’bout . . . But you prefer written this way?
[Ball writes the fraction problem for Brandon both horizontally and vertically:]
\[
\frac{3}{5} + \frac{2}{3} \quad + \frac{2}{3}
\]
Ball: So either way. How would you do that? There the denominators aren’t the same, right?
Brandon: Mm-hmm.
Ball: So what would you do in order to add those?
Brandon: Find the, uh, the smallest number that three and five could go into.
Ball: Okay, so what would that be?
Brandon: Fifteen.
Ball: All right.
[Brandon writes:]
\[
\frac{3 \times 3}{5 \times 3} \quad \frac{9}{15}
\]
\[
+ \frac{2 \times 5}{3 \times 5} \quad \frac{10}{15}
\]
\[
\frac{19}{15}
\]
Ball: Okay. So let’s try a subtraction problem, okay? What if I wrote down four-and-one-third minus two? What would the answer to that be?
[Ball has written:]
\[
\frac{4}{3} - 2 =
\]
Brandon: One-and – one-and-two-thirds?  
Ball: How’d you decide that?  
Brandon: ’Cause minus two . . .

Brandon writes:

Brandon: ’Cause, umm . . . ’Cause it’s – it’s not whole number minus whole number so it – so it – it’s a fraction, so it would . . . So if you minus two, you get two, but it’s – it’s part of one fraction so it – one-and-two-thirds – and two-thirds – the two-thirds – two-thirds plus one-third would equal three.

Ball: I don’t understand where you got the two-thirds from. I don’t see where this came from.

Brandon: ’Cause that plus that would equal two.

Ball: That – this one-and-two-thirds –

Brandon: That’s two-third plus – yeah one-and-two-thirds plus one-and-three-thirds would equal two.

Ball: Okay. And then what?

Brandon: What?

Ball: And then – so then you’re have two. That one – one-and-two-thirds plus one-third equals two.

Brandon: That would equal two, so that would equal a number so that’s why I got that cause it’s different.

Ball: What if you had a fraction that you were subtracting on the bottom as well? So what if you had four-and-one-third minus, umm, two-and-one-half? Then what would you do?

Brandon: Hmm. Find the number that two and third – two and three can go to, ’cause you can’t – you can’t subtract.

Ball: So what would that be?

Brandon: It’d be six.

Ball: Okay.

Brandon: But you don’t do anything to the whole numbers.

Ball: Okay. So now what?

Brandon: And then you subtract.

Ball: Okay, so how would you do that?
Brandon: Umm. Hold on, you can’t subtract yet ’cause two is – two – you can’t subtract two from – you can’t subtract three from two so –

Ball: Mm-hmm

Brandon: – you would cross out four and make it a three.

Ball: Mm-hmm.

Brandon: And would I turn this in – and would you turn this into a twelve?

Ball: How – what are you trying to decide?

Brandon: Umm, cross this out, make the three, and put the whole number right here.

Ball: Okay.

Brandon: And then you subtract.

Ball: Okay, so what would you get?

Brandon: You would get nine – you’d get nine...

Ball: It’s kind of hard to see what you’ve got here. You look like you’ve got twelve-sixths minus three-sixths, and you have three minus two. Do you want me to rewrite it?

Brandon: Yeah.

Ball: Three-and-twelve-sixths, that’s what you have?

Brandon: Mm-hmm.

Ball: And you have, umm, two-and-three-sixths.

[Ball has written:]

\[
\begin{array}{c}
3 \frac{12}{6} \\
2 \frac{3}{6}
\end{array}
\]

Brandon: Hmm. That’s a hard question ’cause if you subtract it, it would be nine – it would be one-and-nine-sixths.

Ball: Yeah.

Brandon: So it’ll be a whole number and an improper fraction.

Ball: Why did you put the one here? I mean I think I have an idea about why you put the one, but I wasn – I’d like to hear you explain it to me.

Brandon: ’Cause –

Ball: Here, I mean.

Brandon: Here – here it – you put three, ’cause three...

Ball: You crossed off the four and made it a three. Then why did you write a one there?
Brandon: Because I borrowed, umm, a whole from the four ’cause you can’t sub – you can’t subtract three from two, so I just crossed out the three, made it – crossed out the four and made it a three and put the one that was taken from the four and put it by the two.

Ball: Okay. So what would you say if I said to you you’re actually working in sixths here, right? So a whole is six-sixths, is that right?

Brandon: Mm-hmm.

Ball: So if you have six-sixths then you wouldn’t be moving a ten over there, right, you’d be moving six-sixths over there. Can you try doing it that way?

Brandon: Mm-hmm.

Ball: And we were trying to subtract two-and-three-sixths?

Brandon: Mm-hmm.

Ball: That’s after you found a common denominator. So what I’m saying is you said you were borrowing a ten, but the whole here would be six-sixths, right? So what if we try again and say you’re going to make that a three and now you’ve got a whole that you can put together with the two-sixths, but what is that whole . . .

Ball: Well you have two-sixths already and now you’re having six more sixths that you got from this one. Now how many six-sixths would you have all together?

Brandon: Umm, six?

Ball: Well you have six-sixths from this whole?

Brandon: Mm-hmm.

Ball: …but you already had two-sixths, so how much is six-sixths plus two-sixths together?

Brandon: One-and-two-sixths?
Ball: Okay, but why don’t you write it as an improper fraction because you’re going to be subtracting. So how much would that be? Six-sixths plus two-sixths?

Brandon: Mm, it would be eight-sixths.

Ball: So write an eight here instead of the two.

[Brandon writes:]

\[ \frac{3}{6} - \frac{2}{3} \]

Ball: Okay, now can you subtract?

Brandon: Uh-huh.

[Brandon writes:]

\[ \frac{3}{6} - \frac{2}{3} = \frac{1}{6} \]

Ball: Okay. So, now walk back ’cause I kind of helped you with that one a little bit. Can you try to explain what I was trying to show you? See if you understand what I was trying to show you?

Brandon: ’Cause –

Ball: Originally what you did is – it seems like you borrowed, like, a ten, the way you normally would borrow, but since we’re working with fractions, I tried to show you a different way to think about the whole. Can you try to explain it back to me?

Brandon: Umm, what you would do is that . . .

Ball: Okay.

Brandon: . . .is that you can’t borrow, like ten, so you would cross, making it eight, so – wait how . . .

Ball: Why did we get the eight from, though?

Brandon: I’m not sure. I’m not sure.

Ball: Okay. Okay. So what we were talking about in part is if you borrow when you’re working with fractions, you have to think what the whole is, right?

Brandon: Six?

Ball: It’s six-sixths, so what we did is we took the six-sixths from this whole and we put it together with the two-sixths we already had. Have you seen that before?
Brandon: Probably, but I probably got confused on it.

Ball: Mm-hmm.

Brandon: So, okay we took — what we — we took the whole from the three and we added it with the two sixths.

Ball: Mm-hmm.

Brandon: So two plus six would be eight —

Ball: Mm-hmm.

Brandon: So it would be eight-sixths.

Ball: Okay. How’re we doing? We’re almost out of time I think. You want to do one more thing, or are you tired?

Brandon: Oh no, we can…

Ball: You’re not tired; you could do this all day? Yeah? Okay? Every time I kid with you they get badly behaved. Okay. Umm.

Ball: So I was thinking of telling you a problem, a story. Have you really ever done anything like that at all, or have you just been working with a lot of numbers and drawings?

Brandon: Mm-hmm.

Ball: Okay. So here’s the story I’m going to tell you, but I’ll have to tell it to you probably even more than one time. Maybe we want some paper to keep track. I’ll tell you the story first. So imagine your teacher, Carol…

Ball: Imagine she bought a cake, but she only bought half a cake. Okay, she didn’t buy a whole cake, she just bought half a cake. Ever see it in the store where they sell half-cakes sometimes? She bought half a cake. Okay? And then she ate half of that. Can you make a picture to show me how much she ate?

Brandon: Mm-hmm.

[Brandon draws:]  

Brandon: This would be half of the cake, so if she ate half of that, she ate half this part.

Ball: Okay. So how much did she eat of the whole cake? If there had been a whole cake, how much would she have eaten of the whole?

Brandon: One-fourth out of it.
Ball: One-fourth of the whole cake, and how much of her cake?
Brandon: Half.
Ball: Half of her cake. Now she was full then . . .
Brandon: Mm-hmm.
Ball: . . . and she decided to split the rest of her cake between you and one of
the other kids in the class, but equally. So this is the part she didn’t eat right
here.

[Ball colors Brandon’s drawing indicating that the shaded portion is what is left
uneaten:]

Ball: Okay, so how could she divide that part equally and give you each an
equal piece?
[Brandon draws:]

Ball: And how much would guys each be getting?
Brandon: What?
Ball: How much of the orig – how much of the whole cake would you each be
getting?
Brandon: Umm, a half of that piece.
Ball: A half of this piece is the remainder, right?
Brandon: Mm-hmm.
Ball: So how much cake is that?
Brandon: Like this?
Ball: Yeah, how much cake is that?
Brandon: Out of the whole – out of this –
Ball: Out of the whole cake.
Brandon: One – I mean one-third. If Carol hadn’t eaten this part then it would
just be one out of third.
Ball: Okay. So you’re getting one-third of her – the cake she bought?
Brandon: Yeah.
Ball: And the other student?

Brandon: Mm-hmm. But since she ate it, since she ate one-third of it then there’s only two-thirds of it remaining.

Ball: But she ate half of it, not a third of it.

Brandon: Oh yeah. So I – I ate and the other friend ate one-half of – the half of her whole cake, or the whole cake

Ball: Right, so what’s one-half of a half?

Brandon: I’m not sure.

Ball: Okay. So you were thinking of these as three equal pieces or three different pieces.

Brandon: Three different pieces.

Ball: Okay. All right. Want to try one more?

Brandon: Mm-hmm.

Ball: Okay. Somebody showed you this problem and said you shouldn’t find a common denominator, or you shouldn’t find an exact answer.

Ball shows Brandon the problem:

\[
\frac{19}{21} + \frac{52}{55}
\]

Brandon: Mm-hmm.

Ball: They just wanted to know about how big is the answer to that. Like is it about – is the total of those two numbers together about one-half? Is the total about one? Is the total about one-and-a-half? About two? What would you say?

Brandon: Umm, I –

Ball: Just approximately.

Brandon: About – about one.

Ball: How did you figure that out?

Brandon: ’Cause – wait no. Actually it would be seventy-out-of – seventy-out-of-eighty, I think. ’Cause since this number is five then I can go up so it would be sixty . . .

Ball: Mm-hmm

Brandon: . . . and this is approximately twenty. And that is approximately twenty, and this is approximately fifty, so fifty plus twenty would be . . .

Ball: You can write it out.

Brandon writes \[
\frac{70}{80}
\]
Brandon: Okay. So this would be seventy and this would be eighty.

Ball: Okay, so if you were to write a fraction that was a whole with twenty-one in the denominator, what would it be?

Brandon: It – to make it a whole?

Ball: Mm-hmm

Brandon: Twenty-one over twenty-one.

Ball: Okay. Is that pretty close to that?

Brandon: Mm-hmm.

Ball: And if you were going to make a fraction that had fifty-five in the denominator that was one, what would you write?

Brandon: Fifty-five.

Ball: Is that pretty close to that?

Brandon: Yeah, but – yeah. Mm-hmm.

Ball: So this one’s pretty close to one and this one’s pretty close to one. When you add them together what’re you going to get?

Brandon: Two.

Ball: You’re going to get pretty close to two. Do you see why?

Brandon: Yeah. ’Cause –

Ball: Can you explain it to me?

Brandon: Yeah, ’cause this is almost one. It’s approximately and this approximately one, so… If we were to round then it – they would be one plus one will, will equal to two.

Ball: Okay. So maybe we should stop. I think we’ve been working a long time and I wanted to ask you a couple questions. What was – of all the stuff we did, and when you look back – we have tons of paper up here now – what do you think was, umm, the most fun that we did, or was anything fun?

Brandon: Yeah. I – I like the number line –

Ball: Uh-huh.

Brandon: – ’cause, umm, I kn – I probably did it few time, but I don’t really work with number lines so it was fun to learn to do something new.

Ball: Uh-huh. So that was pretty new for you?

Brandon: Mm-hmm.

Ball: And of the things we did, what was – what did you find – was there anything that you found difficult that we did?

Brandon: Umm, yeah. Well no, not actually. Mm-hmm.
Brandon: 'Cause I – 'cause probably in the past I probably said, “I don’t re-
member this,” but sort of like if I see it then, yeah I’ll probably remember
or . . . from a long time ago.

Ball: Mm-hmm. Were there things you found that you had to think about, umm,
very hard, that you hadn’t been thinking about for a while?

Brandon: Yeah.

Ball: What – do you remember an example today of something you had to think
really hard about?

Brandon: Umm. About the subtracting – about the subtracting. About, umm,
for – for, umm, like if a number – if – if it’s – if you have a whole number
and a fraction, but the numerator of the fraction can’t subtract the numerator
of the other fraction, you have to borrow from the other one – from the whole.

Ball: Mm-hmm. And you think you had learned that before, or you don’t think
you’ve learned that before?

Brandon: I – yeah I learned it, but – but then I get confused with it, so I use it
a different way –

Ball: Uh-huh.

Brandon: – the incorrect way.

Ball: And how did it feel to have all these people out there? What did you think
about that?

Brandon: Nothing, ’cause we were basically just focusing, like on the board or
on the chalkboard –

Ball: Mm-hmm.

Brandon: – so it wasn’t a big deal.

Ball: Uh-huh. Except when they laughed, of course. Right? Umm, do you have
any questions you wanted to ask me, or anything you wanted to, I don’t know,
anything? Anything about what we did or anything specific?

Brandon: Mm-mm.

Ball: No? Do you want to stop?

Brandon: Mm-hmm.

Ball: So you’re – there’s going to be a little presentation for you, to thank you
for this, so at this point people are going to pay attention to you again. But
the director of the Mathematical Sciences Research Institute – I’m going to
introduce him to you and he’s going to make a little presentation to you. Is
that okay?

Brandon: Mm-hmm.