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The Research Mathematician as Storyteller

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The Southwest Regional Institute in the Mathematical Sciences (SWRIMS) was funded by the National Science Foundation (NSF) as an effort to integrate research and education. Part of the NSF's vision is that the researcher should be simultaneously involved in research activities and educational initiatives. One of the purposes of SWRIMS was to bring into focus what these educational initiatives might be, and to answer the following questions: How can a researcher use research, especially personal achievements in research, to further the educational goals of the mathematics community? How can this research be used to motivate our children to pursue higher mathematical studies? One of us addressed these questions in [5], which catalogued SWRIMS activities for the academic year 1995-96 at its three funded sites (University of Arizona, Northern Arizona University and Utah State University), and formulated some ideas for integrating research and education. The present article aims to highlight the products of one of these activities and to remind the mathematics research community of its teaching role in our society and its responsibility for transmitting our mathematical knowledge to the community.

It is the role of teacher that we have come to re-think as we carried out our SWRIMS activities. This role is very nicely articulated by Robert A. Williams, Jr., in his foreword to *The Rodrigo Chronicles* [2]. Williams, drawing from his background as a member of the Lumbee tribe, states:

> In the Native American tradition, to assume the role of Storyteller is to take on a very weighty vocation. The shared life of a people as a community is defined by an intricate web of connections: kinship and blood, marriage and friendship, alliance and solidarity. In the Indian way, the Storyteller is the one who bears the heavy responsibility for maintaining all of these connections.
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> To be a Storyteller, then, is to assume the awesome burden of remembrance for a people, and to perform this paramount role with laughter and tears, joy and sadness, melancholy and passion, as the occasion demands.

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> There is an art to being a Storyteller, but there is great skill as well. The good Storytellers, the ones who are most listened to and trusted in the tribe, will always use their imagination to make the story fit the occasion.

The stories that SWRIMS would like to tell are stories of relevance, of applicability, of excitement, of importance. They are the stories of our adventures as we forged new tools to address old problems or found new uses for old tools. These adventures have all of the high drama that we would expect from journeys into unexplored territory, and yet we as a profession have kept such adventures hidden. We have not been effective Storytellers in our tribes, and our children have been kept ignorant of this important aspect of our culture. So much so that many in our community state with pride how ignorant they are of mathematical reasoning. We, the research community, should develop these stories and begin to tell them. If we do not, then who will?

In this article we describe two stories that SWRIMS developed for the high schools. NSF mandated that SWRIMS activities should include undergraduate and graduate students, high school and university faculty. Groups with representatives from each category were formed at the various universities. One of the tasks of these groups was to re-package mathematical research in such a way that its results could be understood by a much larger community. It was hoped that performing this task would provide a valuable learning experience for everybody in the group. The researchers would learn how better to communicate mathematics while the high school faculty would better understand the utility of mathematics. When the groups felt that they had developed ideas that would work in a high school classroom, these ideas were tried out in the classroom by everyone in the group.

Why such broadly based groups? For the undergraduate and graduate students in a group, high school was not so long ago. They understand high school student capabilities and anxieties. Their closeness in age adds to their credibility. As Storytellers, their mainstay is the joke placed to give levity when the circumstances become unduly serious or tense.

High school teachers understand the class management issues that university faculty rarely, if ever, consider. For the most part, teachers operate under conditions that researchers would find intolerable and do so with grace, imparting few of the frustrations created by these conditions. As a Storyteller, the teacher's genre may be the anecdote or the riddle, a short piece well placed in the classroom hour that ties stray facts together and starts heads pondering.

Researchers often spend years focused on a specialty among many possible scholarly subjects. We become researchers only after many years of training, and we understand that our achievements can have importance beyond our specialty. The researcher's story is an epic.

We will now tell our epic with jokes, anecdotes, and riddles, "using our imagination to make the story fit the occasion."

First Story: Exponential Growth

Looking back over the two years of the SWRIMS project, we realize how uncertain our beginnings were. We had no fixed ideas on how to proceed and no models from the mathematical community to guide us. As we met with mathematics and biology teachers from several of the Tucson high schools and became acquainted with them and with their programs, our confidence in our ability to make an impact waxed and waned. Part of our early activities had us retreating to a familiar place, the mathematics lecture. The Core Group settled on attending a series of introductory lectures by Jim Cushing, a mathematician whose research focus is mathematical ecology and population dynamics.

Our major goal for the semester was to go to the high school classroom and teach. We had the good fortune to meet the mathematics teaching staff at Sunnyside High School. Their interest, flexibility, and enthusiasm convinced us that Sunnyside was a good place to start. The classroom team — professors Larry Grove, Bill Vélez, and Joe Watkins, high school teachers Doug Cardell, Paul Dye, and Jeff Uecker, and university students Tyler Bayles, Martin Garcia, and Dianna Peña — began holding planning sessions in conjunction with Jim Cushing's lectures. We settled on four full-day visits to Sunnyside, each visit separated by about a week. Our plan was to show exponential growth and decay in a wide variety of contexts — simpler at first, more subtle as we continued. These experiences are summarized in the Southwest RIMS Class Notes titled *The Exponential Function and the Dynamics of Populations*.

Sunnyside is a barrio school on Tucson's south side, with a predominantly Chicano population. The University of Arizona is six miles to the north. Very few of these students have managed to make this six mile journey.

Day 0: Our First Story Begins. The high school teachers invited the Core Group to Sunnyside to see how high school life proceeds. Our intention was to be anonymous observers trying to get a feel for the place. As we would later learn, personal daily greetings are a tradition in the mathematics building at Sunnyside. The students found us quickly and immediately began describing their activity for the day. After much design and construction, they had completed two large paraboloids and were ready to place them at extreme ends of a hallway crowded with students. Could they communicate over the din using the paraboloids to direct sound? They had had a successful test earlier in the day, and they were eager to repeat the test in the Core Group's presence. The second test was an overwhelming success. This is how mathematics instruction proceeds at Sunnyside: mathematical concepts are turned into physical reality by the students themselves.

The typical classroom at Sunnyside has multiple levels — students enrolled in Pre-algebra, Algebra I, Algebra II, and Pre-calculus can regularly be seen working side-by-side. All students study the same topics, but the mathematical techniques and goals are specialized to be appropriate for each student.

Day 1: Powers of Two. Our first aim was to experience the exponential function — both growth and decay — in a variety of ways. To make the case as simply as possible, we began by suggesting activities that generate powers of 2.

Our first exercise was to successively fold a sheet of paper in half. How many sheets thick is the paper after n folds? Some pre-algebra students, not looking for a formula or pattern, counted the sheets after each fold. The advanced students wrote a simple statement: "After n folds, the number of layers is 2^n." The pre-algebra students ask, quite correctly, "What does it mean to have 1024 sheets after 10 folds when I can't even fold the paper in half eight times?" The ensuing discussion give us our first glimpse into the applicability and the drawbacks of a mathematical model.

One student hustled through the exercise, showed us his result, and then asked us, "What mathematics do I need to study if I want to be an economist?" He had been holding this question for a long time, waiting for just the right chance to ask it. We had brought a university catalog for just such an occasion, and went over the program in mathematics and economics at The University of Arizona, where he is now a sophomore.

The second activity uses a probe for measuring distance and a computer to give a graph of the distance between a moving person and the probe. The graphs were displayed on an overhead projector as they were generated. Because the use of a distance probe is likely to be a new experience for many students, we began with a series of warm-up questions and kinesthetic answers to help them become familiar with this technology. The students were slow to start participating, but soon wanted to create their own graphs. Very soon, every student could describe and create a graph using concepts usually introduced in a calculus class.

When the time came to perform the exponential walks, 2^t and 2^{-t}, the students quickly realized that such walks cannot be sustained for any length of time.

One student, whose friend was having a baby, had heard that a fetus doubles in size every month. Using a birth weight of about 8 pounds, we can fill in a table of monthly weights. Guided by this table, the students embarked on a sophisticated discussion of pregnancy. Everyone had been close to at least one classmate who is now a parent, and so the exponential function had meaning for them.

Day 2: Random Models of Growth and Decay. Our second visit to the high school classroom was devoted to more experiences with the exponential function, focusing on models with some random element, so that we no longer had exact powers. Throughout the classroom hour we conducted two activities simultaneously — the growth of bacterial colonies and coin experiments.

One of the undergraduates, Tyler, was the "biologist" in charge of the experiment. During each of the six class hours, he chose a group of laboratory assistants. We grew two colonies of a locally gathered soil bacteria, one at room temperature, and another at the optimal temperature for this strain. We ran the experiment throughout the day, generating a long table of spectrophotometer readings on the blackboard. For some students, this connection between mathematics and biology was the best part of the whole experience, and many would drop by between classes to examine the data.

At the same time that the bacterial growth experiment was proceeding, the students were also performing two experiments, one on growth and one on decay, in which coins were used to simulate simple branching processes with small but positive variance.

Graphs of the experimental results convinced the students that the growth or decay was exponential. Eventually, several students could make a confident conjecture on the growth rate and give an explanation based on an intuitive understanding of the probabilistic mechanism underlying the experiment.

By this time the Core Group knew, through many personal interactions, that Sunnyside was filled with talented and creative students. Many students did not know how remarkable they were. The Sunnyside teachers wasted no opportunities in arranging some unstructured time for the Core Group members to meet with these students.

One Chicana, a truly remarkable young woman, had just returned from a year abroad in Germany and we conversed for a while in German. She excelled in mathematics, but her love was the theater. Her parents had strongly objected to the year abroad; many families feel that a child who physically leaves the community also leaves spiritually. On the other hand, someone returning with gifts like hers can assume a paramount role in this barrio. This was a story of "kinship and blood". Our role was to reinforce this student's sense of self-worth, so that she could gain the resolve to make her own decisions.

Day 3: Population, Plenty, and Poverty. The goal this day was to develop ideas on the purposes of mathematical modeling and gain some understanding of the limitations of models, using human population dynamics. We handed out copies of [3], which gives short introductions to the lives of six families, from Kenya, China, Hungary, India, Brazil, and the United States. We provided context for each story with a summary on each country, obtained from the *CIA World Factbook.*

Many Sunnyside students live in large families, which are important in their lives. So the variety of models for family planning and population growth fueled an interesting discussion, which in turn led the students naturally to the role of mathematical models. What can these models do for us? First of all, in creating a model, we decide what concepts are important and we state our beliefs on the interactions of these concepts. We can use mathematics to study our model and computers to simulate it. We can test our hunches and make informed conjectures. Models can reveal what is important and suggest what to investigate further. We can use models to assess the impact of competing strategies.

We ended by building a mathematical model. Our population would have two age groups — children and adults. The dynamics would be given through a birth rate, a maturation rate, and a death rate. We then proceeded to simulate the model dynamics.

Some of us were designated children, some became adults, and some were consigned to limbo. There was also a census bureau: a "stork" to compute how many babies to bring from limbo, an "escort" to count how many children should become adults, and a "grim reaper" to compute how many adults should die and go to limbo. The grim reaper was never hard to recruit.

Day 4: Models of Populations. Our fourth day was devoted to investigating the model we had developed at the conclusion of Day 3. The rates were chosen so that the eigenvalues for this system were 1 and 1/2. This gave us a constant population total and displayed clearly the role of the eigenvalue 1/2 as the rate of convergence to the eigenvector having eigenvalue 1.

We imagined a large spaceship with 40,000 adults and no children headed for Alpha Centauri and looked at the population through 24 censuses, using either a programmable calculator or Stella, a computer software tool. After working with the data, the students could see that powers of 1 and powers of 1/2 had something to do with this model.

The next question turned out to be quite challenging: If we know the initial population, can we predict the stable population? To address this question, each group picked an initial population of 10,000, variously distributed between children and adults. They reported their stable population — 4000 children and 6000 adults in every case.

Next we varied the initial population sizes and recorded the stable populations. The students still saw no pattern. At this point, the Core Group had a caucus. Eigenvectors, we believed, were a natural concept. How could we get this point across? In a near panic, we suggested that they run the model a third time, and record the stable population in a pie chart.

Ahah! All the pie charts looked the same. So, at Sunnyside High School, eigenvectors are understood as stable pie charts. This experience and the ideas are described in [6].

The students then changed a parameter and were able to predict the stable growth rate and the stable pie chart for their new model.

In this hubbub of activity, we end our first story. We began by folding paper and ended by simulating age structured populations. The Sunnyside students were ready for more. An account of the activities they saw as possible continuations could very well be a list of projects drawn by a researcher in population biology.

An Interlude: Exchanging Stories

The midpoint of two year plan of SWRIMS outreach activities on population dynamics was a conference organized by Jim Cushing of the University of Arizona and Jim Powell of Utah State University. One purpose of this conference was entirely typical — to bring together mathematicians who share research interests and have them report on recent findings. However, this gathering had an additional purpose. The presenters were informed that high school biology and mathematics faculty would be attending, and were asked to prepare their remarks with this larger audience in mind.

On the whole, the lecturers should be praised for their efforts and their successes in communicating their ideas. One researcher remarked: "I am inviting high school teachers to all of my conferences — the extra effort made in preparing the talks made them more accessible and enjoyable for all of us."

Many talks could be adapted to provide an exciting experience for high school students. Particularly noteworthy for our story was the talk presented by James Matis of Texas A&M's Statistics Department. He had developed a probabilistic model for the migration of the Africanized honeybee and used it to make a highly successful prediction of the Africanized honeybee's arrival in the United States through south Texas.

Sunnyside teachers Doug Cardell and Jeff Uecker labeled this talk a "must do" and we arranged for them to talk to Matis and begin preparing his ideas for a high school audience.

Second Story: Bee Population Dynamics

Our second story begins in 1956 in Rio de Janeiro, where a bee researcher hoped he could combine the aggressive characteristics of the African honeybee with the nectar collecting predilections of its European cousin, brought to the Americas in the sixteenth century by the Spaniards, to produce a bee that aggressively collects nectar. During this research, 26 colonies of African honey bees escaped.

Soon after the escape, European and African honeybees began to interbreed, and the resulting *Africanized* honeybees took on the behavior of their African ancestors. Over the succeeding decades, the Africanized bee has been spreading into regions where mean daily temperatures remain above 50°F most of the time. Their migration front has been moving northward at an average speed of 400 miles per year. By 1990, the time and place of their arrival in the United States of America was the subject of scientific research. James Matis gave us the results of his investigations [4].

Joe Watkins, a mathematician in the University of Arizona Core Group, recruited a team to present these research ideas. Undergraduate Tyler Bayles made a return visit to the SWRIMS project. Robert Lanza, a geography graduate student, was brought on the team to work with the bee biologists to design remote sensing maps for use in predicting migration. Rhonda Fleming represented a fine group of environmental biology teachers at Tucson High School.

The tremendous stroke of fortune was the discovery that Tucson hosts one of the five United States Department of Agriculture's bee research centers. Gloria DeGrandi-Hoffman, an entomologist from the Carl Hayden Bee Research Center, works on models for the dynamics of hive populations and on mechanisms for the Africanization of European honeybees [1]. This "bee team" met weekly at the center, from October 1995 to January 1996, to design the activities.

Our goal was to connect many of the broad themes found in a high school environmental biology course to a common topic — the honeybee. For high schools, the novelty of the activities is that mathematical ideas are used to provide insights into biologically meaningful questions. We hoped to intrigue students with current research questions in the biology of bees. The results of the six days of activities are documented in [8]. We wanted to assess our activities with a broad range of students. Thus each day's activities were tested in six different classes at Tucson High School — one regular class, two with at-risk students, two with predominantly Spanish speaking students, and one with many high achieving students — with "bee team" members presenting the activities. We made our visits on consecutive Fridays.

Each day's activities began with a reading. Team members composed these readings by gathering information from beekeepers' handbooks and circulating it around the bee lab for updates and clarifications.

During the class, we discussed the biology of bees extensively, and presented the mathematical activity in clear and concise language. Students formed their own working groups to formulate pertinent questions and to seek out methods of investigation. This method freed the teachers to move among each group of students and to work with them as they addressed the issues and prepared their presentations for the end of the classroom hour. These presentations revealed that many imaginative methods are possible.

These Fridays could be quite intense and the teachers (Oscar Romero, Richard Govern, Kay Wild, and Rhonda Fleming) made careful preparations before and extensive follow-ups after so the day would go well.

Day 1: Sizing up the Population. Working methods for obtaining a reliable census differ dramatically depending on the animal or plant whose population is being tracked. For bees, this can be accomplished by photographing the frames in a hive and estimating the number of bees or brood in the photograph. This task in estimation was the first day's activity. We kept the mathematics gentle because we wanted to develop the idea that mathematics and biology are two subjects that can work side by side.

Day 2: Practical Knowledge of European and Africanized Honey Bees. In the college classroom, the instructor can stick to a syllabus. In high school, the students' demand for a response to all of their concerns is much more pronounced. Some students could not study mathematical models until their doubts about Africanized honeybees were addressed and they were told how to deal with possible stinging incidents. Using materials prepared by the Carl Hayden Bee Research Center, we presented practical training to several hundred students at Tucson High School. In this way, we could advertise our presence and give a context for discussion of bees outside of class.

We addressed the practical side of human co-existence with the honeybee from the viewpoint of the gardener, the pet owner, the outdoor adventurer, the consumer of food, the homeowner, and the beekeeper. Now that a significant fraction of the southern United States is populated by the Africanized honeybee, we must add some scientific knowledge to our practical information.

This training had more relevance than anticipated. One student realized during the presentation that she had encountered a swarm of Africanized bees just to the west of campus. They were safely removed later that day.

Day 3: Hands on Models of Population Dynamics. The exponential and the logistic curves are ubiquitous in describing population growth. On this day, we tried to explain their widespread appearance by describing the elegant mechanisms behind them. We engaged in two activities using pencil, paper, styrofoam cups with lids, and black and white beads to generate exponential and logistic curves.

On this day, we also had to deal with the tragedy of a drive-by shooting. In this case, the victim had nothing to do with the altercation. Students quite naturally had priorities other than playing with beads, recording data in tables, and making graphs. Today a part of our story had tears of sadness and the connections of friendships. The storyteller's skill is needed in respecting this saga of violence and tragedy. We have seen the high school teacher use this skill far too frequently.

Day 4: A Month in the Hive. We asked a lot from the students for this day. Before our arrival, they took their knowledge of bee biology and consolidated it into a flow diagram. Most of the day was dedicated to navigating through the technological apparatus necessary to see how our model predicts the change in a hive population over a month's time.

The effort had its reward. Each pair of students chose a month in the year, a city in the United States, an initial population of brood, house bees, and foragers, and a reasonable guess on the abundance of nectar and pollen. By magic, the three equations used for the model produced a biologically meaningful result over and over again.

This was an entirely new experience for them — mathematical biology had something to it. Concepts from biology could be turned into equations and equations can be used to make predictions. These equations were our best shot at identifying the important concepts. Constructing the model forced us to clarify the relationship between concepts and to choose the most important relationships. The mathematics itself was telling the story.

Day 5: A Year in the Hive. On this day we used a simplified version of BEEPOP, a model produced by the Carl Hayden Bee Research Center, to model a year in the life of a hive. We called our model BEEPOPITA.

This exercise helped students appreciate the effort of ecologists, meteorologists, economists, and epidemiologists in designing and evaluating a model. In addition, we hoped that these students would now be able to evaluate the quality of the predictions they encounter in the popular media. Many models are not based on good science or do not use appropriate mathematical methodology. Many journalists do not have the necessary expertise to make a reliable report of scientific research.

Finding the appropriate balance between mathematics and biology is the continuing challenge in constructing a mathematical model. If the students set out with a model that includes *all* of the concepts in their flow charts, they will be quickly overwhelmed by computation. If their reaction has them removing too many concepts from the biology, the mathematics will be easy. However, the information will not say much about bees (populations, the weather, the economy, or an epidemic).

Day 6: Birth, Death, and Migration. BEEPOP is a model that works well when resources are available, and the hive does not face any circumstances that are overly stressful. These conditions cannot continue forever. At some point, the hive will become overcrowded or will be disturbed. Environmental conditions may change — the hive may be under attack by a bear, by ants, by a pesticide, by disease, or by fire. The queen may no longer be productive. In these instances, the colony must make a critical decision.

The colony of bees can divide or abscond. It can supersede an unproductive queen. If it does nothing, the hive will die.

James Matis, the statistician from Texas A&M, turned the rates at which these critical events happen for Africanized honeybees into a model based on local environmental conditions. Robert Lanza, the geography graduate student on the "bee team", incorporated these model parameters to design a false color composite map of Arizona.

Because the colors are not what the eyes see, some experience with false color composite maps is necessary before they reveal their relevant information. We began our classroom activity by projecting a slide of a false color composite map of the central part of Tucson. After a while, students could obtain information from the map and find significant landmarks in their lives — parks, malls, schools, and their own homes.

Bees and humans differ in their views on which landmarks are important, and so the remote sensing map of Arizona requires a bit more examination before it is useful. However, both bees and humans agree that the rivers, the mountains, the deserts, and the Grand Canyon are important landmarks in Arizona. We also presented a false color composite map of the Mexican state of Sonora. Many Tucsonans have connections to Sonora and know its geography. Also, the Africanized honeybee made its arrival to Arizona via Sonora, and so we can learn about the possibilities of migration in Arizona from the information collected in Sonora.

The birth-death-migration model developed by James Matis and Thomas Kiffe in [4] used the types of models that we had previously seen using beads and styrofoam cups. We added to this model the process of migration — modeled by a random walk. For Matis and Kiffe, this is a two dimensional process. In Arizona, because the movement of the Africanized honey is along riparian areas, the process is one dimensional, and hence easier to simulate.

What are the parameters in our simulation? We can learn this from migration data along the rivers in Sonora. Similar regions in Sonora and Arizona will be revealed by the remote sensing maps. Could we obtain these data? To date, the answer is "no". We know all the ingredients to make the model, but we can not predict when the Africanized honeybee will first reach Hoover Dam until we put our hands on these data.

A Final Note. The bee team met with a purpose: to create a valuable educational experience for high school students. As a consequence, however, the team will also have a significant research achievement.

Gloria had been working on queen development time as an explanation for the Africanization of the honeybee population. By July, 1996, the data were in, but the statistical analysis was not revealing the secrets that the researchers knew were true. In a one hour meeting, one of us (Watkins) and DeGrandi-Hoffman were able to work out the model that established this fact — any place that the Africanized honeybee can live, it will take over. Invasion of species and the inheritance of complex characteristics are basic questions in ecology. This question can now be informed by the example of the Africanization of the honeybee population and the mathematical model that comes with this example.

All of us have faced the embarrassing moment after new acquaintances learn that they are conversing with a mathematician. They feel no connection to us — mathematicians are a distant and foreign breed. Now, when we meet someone, we have stories to tell of friendship, alliance, and solidarity. The response to these stories is always warm. We plan to have more stories to tell and we encourage you to find yours.

Years have passed since we began this activity. Students, beginning with the very first group at Sunnyside, still run to meet us and to tell us how much they enjoyed our visits. By telling stories of mathematics, we set out to change the way people view the world. People's views did change, beginning, most pointedly, with our own.

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