Contemporary Issues in Mathematics Education
MSRI Publications
Volume **36**, 1999

Beyond the Math Wars

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Introduction

Mathematics education is in ferment. Perhaps it always is, but disagreements have become louder; these quarrels have not only captured the general public's attention, but have been exacerbated by articles and opinion columns in the mass media. Almost nothing is considered beyond question, and many people have decided that they are in a "war" in which they need to take sides.

Wars are notable largely for the destruction they wreak. Occasionally they are unavoidable, but there are usually better ways to decide things. This particular war is a war only in the minds of those who wage it, and the supposed sides have more in common with each other than they think. Furthermore, many of the charges made against, or the descriptions of, one supposed side or the other are false.

The purpose of this paper is to puncture some of the myths lying behind the notion that there is a war going on, and to point to areas of broad agreement. Sometimes the agreement is only that a particular issue is an important one, but that is a place to start. Usually the agreement is much deeper.

At the suggestion of the editors, a glossary is provided at the end to describe some of the more technical terms in education. Misunderstanding of what these terms mean is one reason for the notion that war is necessary. Terms in the glossary are in boldface in their first appearance in the text.

This is an expansion of a talk given at the Joint Mathematics Meetings in Baltimore in January, 1998, at a special session co-sponsored by MER, AMS, and MAA. Given the rapid changes since the MSRI meeting in K-12 mathematics education, it seemed preferable to write up this talk rather than the one given over a year earlier at MSRI.

Things are Not What They Seem

The war is generally described as between reform methods and curricula — usually based on the *Standards*[1] published by the National Council for Teachers of Mathematics (NCTM), or somewhat aligned state frameworks and standards — and traditional methods and curricula. But these descriptions are gross oversimplifications. Let me give three examples.

1. Critics of **contextualism** in the schools conflate it with *Standards*-based reform, but the majority of examples in the NCTM *Standards* are not what contextualism recommends, even when they are superficially about something concrete. For example: "I have six coins worth 42 cents ..." No problem beginning with this phrase can be considered contextual in any serious sense.
2. **Saxon**'s books, touted as exemplars of traditional education, are also influenced by the new math movement of the 1960's and 1970's, especially with regard to the emphasis on fairly abstract logical and set-theoretic notions.
3. The technique of scripted **direct instruction**, often touted by opponents of *Standards*-based reform, is decidedly non-traditional. Even more confusingly, of the mathematicians usually identified with reform, a surprisingly large number (I am one of them) became interested in K–12 mathematics education through involvement in **Project SEED** or one of its offspring, which used techniques very close to scripted direct instruction.

Just to show you how messy things are, here are some quotes, taken both off the Web and from published documents. See if you can identify which were uttered by someone associated with reform, and which by someone considered to be opposed to reform.

(i) "Elementary school students should develop rapid facility with addition and multiplication problems."

(ii) "What do students need to memorize? How can that be **facilitated**?"

(iii) "As I looked at the graph, I thought 'Where is what kids are learning? Where is the mathematics?' Nowhere in the display or in the description of the measure of success was there any mention of the mathematics students were studying and what they were learning. This worried me — it worried me a lot.

...The bottom line is that to measure whether a situation has improved, you have to 'show me the mathematics' (to paraphrase the popular saying) — look for the mathematics being taught and learned in the classrooms being observed. The measuring stick should focus ... on what students ... are learning."

[1]There are three volumes, the 1989 *Curriculum and Evaluation Standards*, the 1991 *Professional Standards*, and the 1995 *Assessment Standards*.

(iv) "The duplication of content from year to year in mathematics texts for grades six, seven, and eight is so great that, lacking labels, it is actually difficult to arrange them in the intended order."

(v) "Whatever the reason, proof in the applications, problem-centered domain of secondary school mathematics is postponed — suppressed — downgraded ... There is even strong support for the idea that we should not presume to do much with proof at the secondary level."

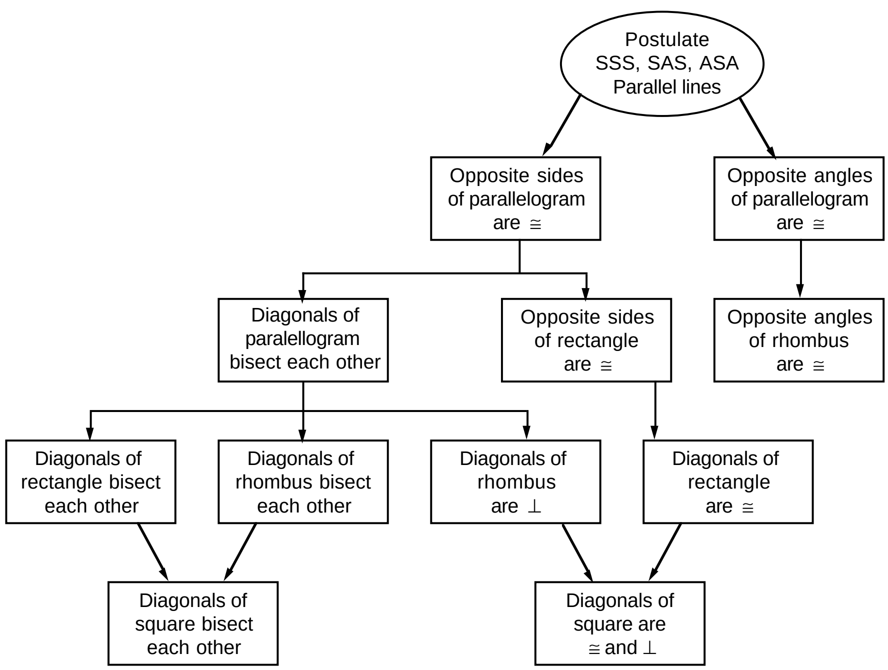
(vi) "... all students can ... deduce properties of, and relationships between, figures from given assumptions."

(vii) "Differences in learning rates must be recognized and provided for."

(viii) "Problems are the life blood of mathematics."

(ix) "... the pressure now exerted by students, administrators and parents to grade on the curve, lower standards, and inflate grades."

(x) And, finally, we have a flowchart:



The first three quotes emphasize rapid facility with arithmetic, memorization, and the need for mathematical substance. Surely they are from one of the documents supporting traditional education. But no, the first two are from the *Professional Standards*, and the third is from an essay by Gail Burrill, President of NCTM, in the NCTM News Bulletin.

Quote 4 and 5 are from a document attacking reform written by Frank Allen,[2] a past president of NCTM who is horrified at the direction NCTM has taken. But it is the traditional texts, not the NSF-funded reform projects, that duplicate content with such gusto, and as for the charge about proof, note that quote 6, which wants all students to give proofs, is from the *Curriculum and Evaluation Standards*.

Quote 7 is problematic. Is it supporting **tracking**? Or **heterogeneous classrooms** that place different demands on different children at the same time in the same place? Since it is from Frank Allen, it is the former, but note how easily the same words could be used to support the latter.

Quote 8 could be contextualism, it could be anything, but it is Frank Allen again, as is quote 9, which charges reform with yet another thing it has nothing to do with. Whatever the origins of grade inflation, *Standards*-based reform is not one of them.

As for the flowchart, it looks like a fairly traditional (except possibly for the format) description of how theorems flow from the postulates of Euclidean geometry, and so it is, embedded in the *Curriculum and Evaluation Standards*.

So things are not what they seem. Out of context it is not easy to figure out who is asking what. Without certain rhetorical phrases that are dead give-aways (the second ellipsis in the third quote leaves out the phrase "all children", generally associated with reform; note that the association itself is a calumny against the traditionalists) it is hard to figure out who is speaking. But since much of the context is political, context can obscure what is being said. You often have to leave the context out to try to figure out what actually is being suggested for the classroom.

Politics Intrude

When politics intrude, what actually is being suggested for the classroom can get lost in the rhetoric. When the politics gets heavy both sides use the same supposed argument. Here is how it goes:

Step 1. Here are my [our] credentials.
Step 2. What you have heard so far is biased.
Step 3. Test results are actually different from what they tell you.
Step 4. What I suggest to you works, and parents know it.
Step 5. Don't be fooled by political lobbying.
Step 6. We must establish world class standards now.

Those familiar with the situation in California, where the state mathematics framework and standards (two different documents, created by nearly independent processes) elicited particularly intense political maneuvering, will recognize

[2]This document was signed by many critics of reform, including mathematicians, and presented at the 1996 NCTM annual meeting as an alternative to the *Standards*.

this list as a description of public testimony given by parents, mathematicians, teachers, and others, before the various boards, commissions, committees, etc., involved in the Byzantine process of shaping the curriculum of California schools. Two things are worthy of note: Steps 1 through 5 are not an argument, and both "sides" used it.

The California situation was most unfortunate in the heavy way it was politicized. Testimony about the framework and standards could take no other form than that described above, since in order to be effective it had to be political (state structure gives final power to the state Board of Education, a politically appointed body with great powers that it does not hesitate to use). The politics in California were more in the public consciousness than in most states; but, thanks to organizations like **HOLD** and **Mathematically Correct**, and the columnist **Debra Saunders**, other states have not been immune to more quiet but equally pervasive politicization of state standards. Comment on state standards in many states tends to be taken out of context and seized on by antagonists in this mythical war, making serious discussion quite difficult. Litmus tests, such as the use of calculators, are rigidly applied. Certain standards (e.g., California's and Virginia's) become battle-cries uttered by people who have no idea what is in them.

The sorts of caricatures that accompany politicization do not lead to helpful solutions to a very serious and long-standing problem, which is that by many measures American schools do not work; in particular, they do not work in math and science. This problem long predates reform. We look lousy in **TIMSS**, but we also looked lousy in **SIMSS** and in **FIMSS**.

Caricatures

These caricatures belie important underlying agreements within the mathematics community.

At the Baltimore panel on which this article is based, Frank Wang, who heads Saxon Publishers, brought up, for his horror-show example, an example very similar to what I was going to bring up. The gist of both of our examples was the same: it isn't that American kids can't calculate (in international comparisons they do well on calculations), it's that they have trouble figuring out what calculations to use if they are not told, and they don't know what the results of the calculations mean. There are two classic examples of this. "Is 10% of 81 $<$, $=$, or $>$ 9?" (American kids often don't know how to begin, even though they do fine when you ask them "What is 10% of 81?") "If you have 147 kids going on a field trip, and at most 36 kids can sit on a school bus, how many buses do you need?" (4 remainder 3, or $4\frac{3}{36}$ are typical answers.)

For another example, within the broad mathematical and mathematics education community, there is agreement that we have largely failed to help teachers learn the mathematics they need in pre-service, and we have largely failed to provide them opportunities to deepen that knowledge throughout their careers. While there is, as there should be, a good deal of debate about what future teachers should learn and how they should learn it, that there needs to be a sharp increase in teacher's mathematical knowledge is denied by no-one, and the many interesting experiments and proposals that exist cannot easily be categorized by the terms used to describe protagonists in the mythical math wars.

Even when it comes down to specifics, there is a surprising amount of agreement — everyone loves the way the Japanese teach mathematics and wants to claim them for their own; the Singapore framework and the Russian problem books have a broad range of admirers.

There are serious issues on which people differ, but how they differ on those issues does not arrange itself neatly.

What are the caricatures that keep us from seeing the problem and the areas of agreement?

Here are some of the false charges made against the *Standards*:

(i) No conventional algorithms are to be taught.
(ii) **Constructivism** rules: children must invent all of school mathematics.
(iii) Individual work is discouraged: children must not only invent all of school mathematics, but they must invent it solely by working in small groups.
(iv) The concept of "proof" is essentially eliminated.
(v) Facility with arithmetic and algebraic manipulations is discouraged.
(vi) Mistakes go uncorrected — everything is okay in **"fuzzy math."**
(vii) Contextualism rules: all of school mathematics must be motivated by real-world problems.
(viii) Teachers never lecture, they facilitate.

Here are some of the false charges made against critics of the *Standards* (i.e., traditionalists):

(i) Only conventional algorithms are allowed.
(ii) Children must do as they are told.
(iii) Only individual work is allowed — worksheet after worksheet after worksheet, with no chance for class discussion.
(iv) "Proof" means dry and dull two-column proofs in geometry of obvious statements from other obvious statements; no other reasoning is encouraged.
(v) Except for Euclidean geometry (see #4), only facility with arithmetic and algebraic manipulations is the focus of the curriculum.
(vi) Mistakes are to be corrected immediately by the teacher, without discussion.
(vii) The only word problems allowed are those for which the teacher can present a precise algorithm for solution.
(viii) Teachers only talk at students; teachers never listen to students and are insensitive to student needs.

These statements have just enough grounding in truth (it doesn't take very much) to be believable: for example, children in traditional classrooms often spend a lot of time working on worksheets, and children learning from NSF-sponsored curricula often spend a significant amount of time working in small groups. But they are also caricatures: good teachers of all sorts involve their classes in discussion, and children learning from NSF-sponsored curricula also spend a significant amount of time working on their own.

When discussion focuses on these caricatures, we become sidetracked. They are all false, and that is all that needs to be said about them. Instead of being distracted, we should get down to business.

Why Can't We Get Down to Business?

Roger Howe, a professor of mathematics at Yale University, and former chair of the AMS **ARG** (see glossary), has listed key issues in mathematics education in [2]. Here is his list:

(i) relative performance (in international comparisons)
(ii) equity
(iii) technology
(iv) demography
(v) subject matter
(vi) pedagogy
(vii) teacher preparation and certification
(viii) assessment
(ix) high performers
(x) new curricula

This is the business we should be getting down to, in a civil and professional manner. But there are at least four delusions that make it difficult to discuss these issues.

The delusion of assessment. This is the notion that there is a way to find out which pedagogical method or curriculum works.

Ron Ferguson, a professor of in the Department of Mathematics and Computer Science at Texas A&M, clearly delineated the problems of designing a fail-safe study in a message sent to the math-teach e-mail list (run by Gene Klotz' estimable Math Forum, an excellent source for references in mathematics education; see http://forum.swarthmore.edu/). He considered a simple situation: design a study of calculator use in second grade, taking into account interaction with teaching strategies. What school district wouldn't love such a study?

Here are the pitfalls Ron pointed out: First you must randomly select teachers to take part — but some won't do it. Then you have to randomly assign both calculator/non-calculator use and teaching method — but some teachers will refuse to teach by the assigned method, or refuse/insist on using calculators;

to do otherwise would go against their basic beliefs about teaching. Then you have to carefully coach the method. Then you have to carefully monitor what goes on in the classroom — no deviation can be permitted. Only after all this can you collect data (and my question is: which data do you collect?). Finally you do the appropriate statistical analysis. And even then — so what? How does what happens in second grade affect what happens in ninth grade? We may theorize, but do we actually know? And even if we did — so what? The results are statistical, they don't tell us what will work with this kid right now. And even if we knew that, we would be left with a philosophical question: do students need to be able to do addition without the aid of a calculator?

To quote Ron directly: "There is nothing quite so violent as a war based on differences in faith ... Good teaching ... is a day by day experiment in which the teacher tries to find a combination of new and old that works with some student."

This is not to say that there is nothing to be learned from research in mathematics education. There are a great many pieces of wisdom to learn. But what is "best" is not one of them.

The delusion that curriculum can be judged on the page. We need serious discussion of curriculum — from mathematicians, from teachers, from people who do research in mathematics education. The ARG reports have given us a good start on issues such as proof/reasoning/logic; algorithm; algebra and precursors to algebra; mathematical modeling/word problems; statistics. The ARG discussions have been refreshingly free of ideology: one does not argue for the importance in school mathematics of the geometric series or transformational geometry or the study of algorithms *qua* algorithms because one is a traditionalist or a *Standards*-based reformer. But within the mathematics community there is a tendency to judge on the page and not in the classroom. I have seen materials that I thought were terrible work well in a classroom; and we all have the experience of preparing mathematically elegant material only to discover that our students didn't understand one bit of it. Our judgments should be tempered by the knowledge that what is on the page is only a small part of what happens in the classroom.

The delusion that, without a lot of observation in a variety of classrooms, we still know what goes on in the schools. We don't. What happened to us n years ago or what is happening to our child right now is only a small section of a highly heterogeneous solid.

The delusion that if we want to fix it they will let us, and even pay for it. Mathematicians are not particularly welcome at many of the relevant tables, being seen variously as superfluous or arrogant. There are signs of improvement (for example, the ARGs, or the fact that six of the 24 members of the NCTM *Standards* revision writing team have done significant mathematical research), but the general statement remains.

At the MSRI meeting which is the occasion for this volume I presented my own home-made unscientific chart of who has power in mathematics education. State legislators and other politically related sorts (e.g., state school boards) were at the top; teachers were near the bottom, and mathematicians were at the very bottom.

Even within the universities resources are lacking for the mathematics education of teachers, the one aspect of K–12 education for which everyone agrees mathematicians bear some reponsibility. There is general agreement that what we do now is inadequate, and even the beginnings of an outline of what teachers should know. Hung-Hsi Wu, a professor of mathematics at Berkeley, most conveniently has an article in this volume on this issue [5] which provides one place to start, and one of the few good things to have come out of the California situation is his involvement in a relatively well-funded attempt to work with teachers. (He has an interesting preprint describing the mathematics he observed in existing teacher enhancement projects [6].) Al Cuoco, Paul Goldenberg and their colleagues at **EDC** have produced material which provides another, not incompatible, place to start (e.g., [1]), and their EDC colleague Deborah Schifter has produced interesting volumes on teacher enhancement [3; 4]. (These references are by no means comprehensive, but simply a place to begin.) The U.S. Department of Education is funding an MAA/CBMS grant to examine the issue of teacher preparation. Discussion is going on through e-mail lists, within departments, at meetings. But what is missing is institutional will. Rather than being considered a major part of the educational mission, in many universities teacher preparation is relegated far below engineering education, and teacher enhancement isn't part of the official mission at all. One of the major tasks we face is convincing our colleagues, our deans, our local school districts, and our state departments of education to change this situation.

A Cautionary Tale

Recently, on an e-mail list, there was a very long and at times vituperative debate about two versions of a problem about coins. Version 1 is "I have a nickel, a dime, and a penny. How much money do I have?" Version 2 is "I have three coins. How much money could I have?"

One group essentially said: Version 2 is terrible! What sort of answer will a teacher expect? How can a kid learning how to add 1's, 5's, 10's, and 25's be expected to solve it? They will find one or maybe two possibilities and then think they're done!

Another group essentially said: Version 2 is great! It encourages kids to list possibilities carefully! It lets kids practice their arithmetic skills! It encourages them to check other kids' answers!

Ten years ago someone might have said Version 1 is no good, but we've made enough progress so that at least that didn't happen.

Now what was wrong with this debate is that several weeks went by before it became clear what the debate was really about. The issue wasn't whether either of these problems are good or bad (they both are good.) The issue is: do you teach basic arithmetic first and then other mathematical skills later? Or do you introduce skills that will be important later, like listing things systematically, *while* kids are learning basic arithmetic? And if the latter, what is appropriate when? But until the issue was clear, we were talking past each other.

The issue was not clear because version 2 is clearly a reform-type problem. A more specific version (where we know the coins are among pennies, nickels and dimes) shows up in the *Curriculum and Evaluation Standards*, and the contrast between the two versions was actually suggested by NCTM leadership to the media to show the difference between traditional and reform problems. Version 2 thus carried a political burden — attitudes about reform became projected onto it, and it was very difficult to clearly see what we were talking about. I do not exempt myself from the influence of politics; my own early comments on it were unduly sunny (I thought first graders could handle four possible coins).

Summary

Let me sum up this paper in four sentences: There is no math war. Politicization distorts things. We have too much to work on to spend time sniping at each other. We need to get down to business.

Glossary

ARG: Association Resource Group. These are committees set up by various organizations, including many of the mathematical organizations, to assist the NCTM in revising the *Standards*. For information on the various ARG reports, see the relevant society journals and Web pages, as well as the NCTM Web page.

Constructivism: 1. The belief that knowledge is necessarily constructed, not passively received. 2. Often misinterpreted to mean that children should reinvent all of mathematics. 3. Often misconstrued (under both the rubrics of reform and of anti-reform) as a pedagogical method rather than an epistemological belief. This comes from the fact that constructivism does have pedagogical consequences. These consequences are not, as is commonly believed, a rejection of all lecturing in favor of children working in small groups, but rather come from paying careful attention to the question "How can I present this material so my students can make sense of it?" The phrase "make sense" (emphasis on "make") instead of "learn" is what makes this a constructivist question. 4. Sometimes confused with social constructivism (the belief that knowledge is socially constructed).

Contextualism: The belief that all of the mathematics we teach our students should be relevant to their lives.

Direct instruction: A pedagogical method similar to group discovery (see Project SEED), with the addition of periods of rote recitation (e.g., of arithmetic facts).

EDC: Education Development Corporation, an education think tank spun off from MIT.

Facilitate: 1. The sort of word a constructivist teacher might use. 2. Sometimes misconstrued as an invitation for the teacher to provide no direction.

FIMSS: First International Mathematics and Science Study. In the U.S., run by the U.S. Department of Education and the National Center for Education Statistics.

Fuzzy math: One of the pejorative terms (another is "New New Math") used by opponents of *Standards*-based reform to describe *Standards*-based reform.

Heterogeneous classroom: A classroom with children of varying abilities and skills.

Hirsch: 1. E. D. Hirsch, A college English professor, author of *The schools we need and why we don't have them.* 2. A list of precise curriculum goals, subject by subject and grade by grade, associated with Hirsch's criticism of our schools.

HOLD: An anti-reform parent group in Palo Alto. See http://www.rahul.net/dehnbase/hold/.

Homogeneous classroom: A classroom in which children have a narrow range of abilities and skills.

Mathematically Correct: An anti-reform parent group based in Southern California, with members all over the country. See http://ourworld.compuserve.com/homepages/mathman/.

Project SEED: A program founded in the 1970's that brought group discovery learning to elementary children in underprivileged neighborhoods. Group discovery as envisioned by SEED's founder, Bill Johntz, was scripted. The teacher (generally not a regular teacher, often a mathematics or education graduate student) prepared a sequence of questions, the answer to each being short and fairly obvious, but the entire sequence designed to lead children through fairly advanced topics, e.g., negative numbers in second grade.

Saunders, Debra: A columnist in California who generally does not like what is going on in the schools.

Saxon: 1. John Saxon, a retired army officer who wrote a series of mathematics texts and founded a publishing company to produce and sell them. 2. The Saxon textbook series. The main principles adopted by Saxon are: learning takes place in small increments; problems from earlier sections should occur in later sections; students need a lot of problems to practice on; explanation should be kept short; problems should be close to template problems.

SIMSS: Second International Mathematics and Science Study. In the U.S., it was run by the U.S. Department of Education and the National Center for Education Statistics.

TIMSS: Third International Mathematics and Science Study. In the U.S., it was run by the U.S. Department of Education and the National Center for Education Statistics.

Tracking: placing children in homogeneous classrooms.

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