

## THE HOMOLOGICAL CONJECTURES

MSRI Hot Topics Workshop

March 12 to 16, 2018

### BOOKS

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#### OTHER RESOURCES

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