

# NEW GEOMETRIC TECHNIQUES IN NUMBER THEORY

MSRI Summer Graduate School

July 1–12, 2013

- [BLGG13] T. BARNET-LAMB, T. GEE, and D. GERAGHTY, Serre weights for rank two unitary groups, *Math. Ann.* (2013). doi: 10.1007/s00208-012-0893-y.
- [BLGGT13] T. BARNET-LAMB, T. GEE, D. GERAGHTY, and R. TAYLOR, Potential automorphy and change of weight, *Ann. of Math. (2)* (2013), To appear. arXiv 1010.2561.
- [BLGHT11] T. BARNET-LAMB, D. GERAGHTY, M. HARRIS, and R. TAYLOR, A family of Calabi–Yau varieties and potential automorphy II, *Publ. Res. Inst. Math. Sci.* **47** (2011), no. 1, 29–98. MR 2827723. doi: 10.2977/PRIMS/31.
- [Bö13] G. BÖCKLE, Deformations of Galois representations, in *Elliptic curves, Hilbert modular forms and Galois deformations* (H. DARMON, F. DIAMOND, L. V. DIEULEFAIT, B. EDIXHOVEN, and V. ROTGER, eds.), Birkhäuser, Basel, 2013, pp. 21–115. doi: 10.1007/978-3-0348-0618-3\_2.
- [BT05] F. BOGOMOLOV and Y. TSCHINKEL (eds.), *Geometric methods in algebra and number theory*, Progress in Mathematics **235**, Birkhäuser, Boston, 2005, ISBN 0-8176-4349-4. MR 2159374. Zbl 1076.11001. doi: 10.1007/b138649.
- [BD13] C. BREUIL and F. DIAMOND, *Formes modulaires de Hilbert modulo  $p$  et valeurs d’extension Galoisiennes*, Preprint, 2013. arXiv 1208.5367.
- [BC09] O. BRINON and B. CONRAD, *CMI summer school notes on  $p$ -adic Hodge theory*, Preliminary version, 2009. Available at <http://math.stanford.edu/~conrad/papers/notes.pdf>.
- [Bum97] D. BUMP, *Automorphic forms and representations*, Cambridge Studies in Advanced Mathematics **55**, Cambridge Univ. Press, Cambridge, 1997, ISBN 0-521-55098-X. MR 1431508. Zbl 0868.11022. doi: 10.1017/CBO9780511609572.
- [BFG12] D. BUMP, S. FRIEDBERG, and D. GOLDFELD (eds.), *Multiple Dirichlet series,  $L$ -functions and automorphic forms*, Progress in Mathematics **300**, Birkhäuser, New York, 2012, ISBN 978-0-8176-8333-7. MR 2961902. Zbl 1257.11001. doi: 10.1007/978-0-8176-8334-4.
- [Buz12] K. BUZZARD, Potential modularity: a survey, in *Non-abelian fundamental groups and Iwasawa theory* (J. COATES, M. KIM, F. POP, M. SAIDI, and P. SCHNEIDER, eds.), London Math. Soc. Lecture Note Ser. **393**, Cambridge Univ. Press, Cambridge, 2012, pp. 188–211. MR 2905534. Zbl 06059398. arXiv 1101.0097. doi: 10.1017/CBO9780511984440.008.
- [CG12a] F. CALEGARI and D. GERAGHTY, *Modularity lifting beyond the Taylor–Wiles method*, Preprint, 2012. arXiv 1207.4224.
- [CG12b] F. CALEGARI and D. GERAGHTY, *Modularity lifting beyond the Taylor–Wiles method II*, Preprint, 2012. arXiv 1209.6293.
- [Car94] H. CARAYOL, Formes modulaires et représentations Galoisiennes à valeurs dans un anneau local complet, in  *$p$ -adic monodromy and the Birch and Swinnerton-Dyer conjecture* (Boston, 1991), Contemp. Math. **165**, Amer. Math. Soc., Providence, RI, 1994, pp. 213–237. MR 1279611. Zbl 0812.11036. doi: 10.1090/conm/165/01601.
- [CF67] J. W. S. CASSELS and A. FRÖHLICH (eds.), *Algebraic number theory*, Academic Press, London, 1967. MR 0215665. Zbl 0153.07403.
- [Cho09] S.-H. CHOI, *Local deformation lifting spaces of mod  $l$  Galois representations*, PhD thesis, Harvard University, Cambridge, MA, 2009. Available at <http://gradworks.umi.com/33/65/3365229.html>.

- [Clo90] L. CLOZEL, *Motifs et formes automorphes: applications du principe de fonctorialité*, in *Automorphic forms, Shimura varieties, and L-functions, I* (Ann Arbor, MI, 1988), *Perspect. Math.* **10**, Academic Press, Boston, 1990, ISBN 0-12-176651-9, pp. 77–159. MR 1044819. Zbl 0705.11029.
- [CHT08] L. CLOZEL, M. HARRIS, and R. TAYLOR, Automorphy for some  $l$ -adic lifts of automorphic mod  $l$  Galois representations, *Publ. Math. Inst. Hautes Études Sci.* (2008), no. 108, 1–181. MR 2470687. Zbl 1169.11020. doi: 10.1007/s10240-008-0016-1.
- [CM90] L. CLOZEL and J. S. MILNE (eds.), *Automorphic forms, Shimura varieties, and L-functions, I* (Ann Arbor, MI, 1988), *Perspect. Math.* **10**, Academic Press, Boston, 1990, ISBN 0-12-176651-9. MR 1044817. Zbl 0684.00003. Available at <http://www.jmilne.org/math/Books/AA1988a.pdf>.
- [CGMS89] J. COATES, R. GREENBERG, B. MAZUR, and I. SATAKE (eds.), *Algebraic number theory* (Berkeley, CA, 1987), *Advanced Studies in Pure Mathematics* **17**, Academic Press, Boston, 1989, ISBN 0-12-177370-1. MR 1097604. Zbl 0721.00006.
- [CKP<sup>+</sup>12] J. COATES, M. KIM, F. POP, M. SAIDI, and P. SCHNEIDER (eds.), *Non-abelian fundamental groups and Iwasawa theory*, *London Math. Soc. Lecture Note Ser.* **393**, Cambridge Univ. Press, Cambridge, 2012, ISBN 978-1-107-64885-2. MR 2895642. Zbl 1237.11001. doi: 10.1017/CBO9780511984440.
- [CY97] J. COATES and S. T. YAU (eds.), *Elliptic curves, modular forms & Fermat's last theorem* (Hong Kong, 1993), *Int. Press*, Cambridge, MA, 1997, ISBN 1-57146-049-7. MR 1605709. Zbl 0997.11504.
- [CSS97] G. CORNELL, J. H. SILVERMAN, and G. STEVENS (eds.), *Modular forms and Fermat's last theorem* (Boston, 1995), *Springer*, New York, 1997, ISBN 0-387-94609-8; 0-387-98998-6. MR 1638473. Zbl 0878.11004. doi: 10.1007/978-1-4612-1974-3.
- [CLO07] D. COX, J. LITTLE, and D. O'SHEA, *Ideals, varieties, and algorithms: an introduction to computational algebraic geometry and commutative algebra*, 3rd ed., *Springer*, New York, 2007, ISBN 978-0-387-35650-1; 0-387-35650-9. MR 2290010. Zbl 1118.13001. doi: 10.1007/978-0-387-35651-8.
- [CR06] C. W. CURTIS and I. REINER, *Representation theory of finite groups and associative algebras*, *AMS Chelsea*, Providence, RI, 2006, Reprint of the 1962 original. ISBN 0-8218-4066-5. MR 2215618. Zbl 1093.20003.
- [DDT97] H. DARMON, F. DIAMOND, and R. TAYLOR, Fermat's last theorem, in *Elliptic curves, modular forms & Fermat's last theorem* (Hong Kong, 1993) (J. COATES and S. T. YAU, eds.), *Int. Press*, Cambridge, MA, 1997, pp. 2–140. MR 1605752. Zbl 0877.11035.
- [D<sup>+</sup>66] S. P. DEMUSHKIN and OTHERS, *Twelve papers on algebra, number theory and topology*, *Amer. Math. Soc. Transl. (2)* **58**, *Amer. Math. Soc.*, Providence, RI, 1966.
- [Dia97] F. DIAMOND, The Taylor–Wiles construction and multiplicity one, *Invent. Math.* **128** (1997), no. 2, 379–391. MR 1440309. Zbl 0916.11037. doi: 10.1007/s002220050144.
- [DT94] F. DIAMOND and R. TAYLOR, Lifting modular mod  $l$  representations, *Duke Math. J.* **74** (1994), no. 2, 253–269. MR 1272977. Zbl 0809.11025. doi: 10.1215/S0012-7094-94-07413-9.
- [Dic01] M. DICKINSON, On the modularity of certain 2-adic Galois representations, *Duke Math. J.* **109** (2001), no. 2, 319–382. MR 1845182. Zbl 1015.11020. doi: 10.1215/S0012-7094-01-10923-X.
- [Far06] L. FARGUES, *Motives and automorphic forms: the (potentially) abelian case*, *Delivered at IHES-Orsay Summer School*, 2006. Available at [http://www.math.jussieu.fr/~fargues/Motifs\\_abeliens.pdf](http://www.math.jussieu.fr/~fargues/Motifs_abeliens.pdf).
- [Fon94] J.-M. FONTAINE, Le corps des périodes  $p$ -adiques, *Astérisque* **223** (1994), 59–111. MR 1293971. Zbl 0940.14012.
- [FL82] J.-M. FONTAINE and G. LAFFAILLE, Construction de représentations  $p$ -adiques, *Ann. Sci. École Norm. Sup. (4)* **15** (1982), no. 4, 547–608 (1983). MR 707328. Zbl 0579.14037. Available at [http://www.numdam.org/item?id=ASENS\\_1982\\_4\\_15\\_4\\_547\\_0](http://www.numdam.org/item?id=ASENS_1982_4_15_4_547_0).
- [FM95] J.-M. FONTAINE and B. MAZUR, Geometric Galois representations, in *Elliptic curves, modular forms, & Fermat's last theorem* (Hong Kong, 1993) (J. COATES and S. T. YAU, eds.), *Int. Press*, Cambridge, MA, 1995, pp. 41–78. MR 1363495. Zbl 0839.14011.

- [Gee13] T. GEE, *Modularity lifting theorems*, Based partly on Richard Taylor's 2009 course at Harvard, 2013. Available at [http://www2.imperial.ac.uk/~tsg/Index\\_files/ArizonaWinterSchool2013.pdf](http://www2.imperial.ac.uk/~tsg/Index_files/ArizonaWinterSchool2013.pdf).
- [GLS09a] D. GINZBURG, E. LAPID, and D. SOUDRY (eds.), *Automorphic forms and L-functions, I: Global aspects* (Rehovot and Tel-Aviv, 2006), Contemp. Math. **488**, Amer. Math. Soc., Providence, RI, 2009, ISBN 978-0-8218-4706-0. MR 1500286. Zbl 1166.11002. doi: 10.1090/conm/488.
- [GLS09b] D. GINZBURG, E. LAPID, and D. SOUDRY (eds.), *Automorphic forms and L-functions, II: Local aspects* (Rehovot and Tel-Aviv, 2006), Contemp. Math. **489**, Amer. Math. Soc., Providence, RI, 2009, ISBN 978-0-8218-4708-4. MR 2531715. Zbl 1167.11002. doi: 10.1090/conm/489.
- [Gou88] F. Q. GOUVÊA, *Arithmetic of p-adic modular forms*, Lecture Notes in Mathematics **1304**, Springer, Berlin, 1988, ISBN 3-540-18946-7. MR 1027593. Zbl 0641.10024. doi: 10.1007/BFb0082111.
- [IRS89] Y. IHARA, K. RIBET, and J.-P. SERRE (eds.), *Galois groups over  $\mathbf{Q}$*  (Berkeley, CA, 1987), Mathematical Sciences Research Institute Publications **16**, Springer, New York, 1989, ISBN 0-387-97031-2. MR 1012165. Zbl 0684.00005. doi: 10.1007/978-1-4613-9649-9.
- [KKS00] K. KATO, N. KUROKAWA, and T. SAITO, *Number theory, 1: Fermat's dream*, Translations of Mathematical Monographs **186**, Amer. Math. Soc., Providence, RI, 2000, ISBN 0-8218-0863-X. MR 1728620. Zbl 0953.11003.
- [KW09] C. KHARE and J.-P. WINTENBERGER, Serre's modularity conjecture, II, *Invent. Math.* **178** (2009), no. 3, 505–586. MR 2551764. Zbl 05636296. doi: 10.1007/s00222-009-0206-6.
- [Kis08] M. KISIN, Potentially semi-stable deformation rings, *J. Amer. Math. Soc.* **21** (2008), no. 2, 513–546. MR 2373358. Zbl 1205.11060. doi: 10.1090/S0894-0347-07-00576-0.
- [Kis09a] M. KISIN, The Fontaine–Mazur conjecture for  $GL_2$ , *J. Amer. Math. Soc.* **22** (2009), no. 3, 641–690. MR 2505297. Zbl 1251.11045. doi: 10.1090/S0894-0347-09-00628-6.
- [Kis09b] M. KISIN, Modularity of 2-adic Barsotti–Tate representations, *Invent. Math.* **178** (2009), no. 3, 587–634. MR 2551765. Zbl 05636297. doi: 10.1007/s00222-009-0207-5.
- [Kis09c] M. KISIN, Moduli of finite flat group schemes, and modularity, *Ann. of Math. (2)* **170** (2009), no. 3, 1085–1180. MR 2600871. Zbl 1201.14034. doi: 10.4007/annals.2009.170.1085.
- [Kna92] A. W. KNAPP, *Elliptic curves*, Mathematical Notes **40**, Princeton Univ. Press, Princeton, NJ, 1992, ISBN 0-691-08559-5. MR 1193029. Zbl 0804.14013.
- [KSY08] T. KOBAYASHI, W. SCHMID, and J.-H. YANG (eds.), *Representation theory and automorphic forms* (Seoul, 2005), Progress in Mathematics **255**, Birkhäuser, Boston, 2008, ISBN 978-0-8176-4505-2. MR 2369547. Zbl 1124.11004. doi: 10.1007/978-0-8176-4646-2.
- [Kob82] N. KOBLITZ (ed.), *Number theory related to Fermat's last theorem* (Cambridge, MA, 1981), Progress in Mathematics **26**, Birkhäuser, Boston, 1982, ISBN 0-7643-3104-6. MR 685284. Zbl 0491.00009.
- [Kob93] N. KOBLITZ, *Introduction to elliptic curves and modular forms*, 2nd ed., Graduate Texts in Mathematics **97**, Springer, New York, 1993, ISBN 0-387-97966-2. MR 1216136. Zbl 0804.11039. doi: 10.1007/978-1-4612-0909-6.
- [Kob94] N. KOBLITZ, *A course in number theory and cryptography*, 2nd ed., Graduate Texts in Mathematics **114**, Springer, New York, 1994, ISBN 0-387-94293-9. MR 1302169. Zbl 0819.11001. doi: 10.1007/978-1-4419-8592-7.
- [Koc92] H. KOCH, *Number theory, II: Algebraic number theory*, Encyclopaedia of Mathematical Sciences **62**, Springer, Berlin, 1992, ISBN 3-540-53386-9. MR 1218887. Zbl 0819.11044. doi: 10.1007/978-3-642-58095-6.
- [Koc00] H. KOCH, *Number theory: algebraic numbers and functions*, Graduate Studies in Mathematics **24**, Amer. Math. Soc., Providence, RI, 2000, ISBN 0-8218-2054-0. MR 1760632. Zbl 0953.11001.
- [Kol95] J. KOLLÁR, *Shafarevich maps and automorphic forms*, Princeton Univ. Press, Princeton, NJ, 1995, ISBN 0-691-04381-7. MR 1341589. Zbl 0871.14015.

- [Kum75] E. E. KUMMER, *Collected papers, I: Contributions to number theory*, Springer, Berlin, 1975. MR 0465760. Zbl 0327.01019.
- [Maz89] B. MAZUR, Deforming Galois representations, in *Galois groups over  $\mathbf{Q}$*  (Berkeley, CA, 1987), Math. Sci. Res. Inst. Publ. **16**, Springer, New York, 1989, pp. 385–437. MR 1012172. Zbl 0714.11076. doi: 10.1007/978-1-4613-9649-9\_7.
- [Maz97] B. MAZUR, An introduction to the deformation theory of Galois representations, in *Modular forms and Fermat's last theorem* (Boston, 1995) (G. CORNELL, J. H. SILVERMAN, and G. STEVENS, eds.), Springer, New York, 1997, pp. 243–311. MR 1638481. Zbl 0901.11015.
- [Mil06] J. S. MILNE, *Arithmetic duality theorems*, 2nd ed., BookSurge, Charleston, SC, 2006, ISBN 1-4196-4274-X. MR 2261462. Zbl 1127.14001. Available at <http://www.jmilne.org/math/Books/adt.html>.
- [Neu99] J. NEUKIRCH, *Algebraic number theory*, Grundlehren der Mathematischen Wissenschaften **322**, Springer, Berlin, 1999, ISBN 3-540-65399-6. MR 1697859. Zbl 0956.11021. doi: 10.1007/978-3-662-03983-0.
- [Pil08] V. PILLONI, *The study of 2-dimensional  $p$ -adic Galois deformations in the  $l \neq p$  case*, Preprint, 2008. Available at <http://perso.ens-lyon.fr/vincent.pilloni/Defo.pdf>.
- [Ram93] R. RAMAKRISHNA, On a variation of Mazur's deformation functor, *Compositio Math.* **87** (1993), no. 3, 269–286. MR 1227448. Zbl 0910.11023. Available at [http://www.numdam.org/item?id=CM\\_1993\\_\\_87\\_3\\_269\\_0](http://www.numdam.org/item?id=CM_1993__87_3_269_0).
- [Sat85] I. SATAKE (ed.), *Automorphic forms and number theory* (Sendai, 1983), Advanced Studies in Pure Mathematics **7**, North-Holland, Amsterdam, 1985, ISBN 0-444-87940-4. MR 876098. Zbl 0597.00007.
- [Ser94] J.-P. SERRE, *Cohomologie Galoisienne*, 5th ed., Lecture Notes in Mathematics **5**, Springer, Berlin, 1994, ISBN 3-540-58002-6. MR 1324577. Zbl 0812.12002.
- [Ser04] J.-P. SERRE, *Corps locaux*, 4th ed., Publications de l'Université de Nancago **8**, Hermann, Paris, 2004, Translated as *Local fields*, Graduate Texts in Mathematics **67**, Springer, New York, 1979. MR 0354618. Zbl 1095.11504.
- [Shi68] G. SHIMURA, *Automorphic functions and number theory*, Lecture Notes in Mathematics **54**, Springer, Berlin, 1968. MR 0234915. Zbl 0183.25402.
- [Shi00] G. SHIMURA, *Arithmeticity in the theory of automorphic forms*, Mathematical Surveys and Monographs **82**, Amer. Math. Soc., Providence, RI, 2000, ISBN 0-8218-2671-9. MR 1780262. Zbl 0967.11001.
- [SW99] C. M. SKINNER and A. J. WILES, Residually reducible representations and modular forms, *Inst. Hautes Études Sci. Publ. Math.* **89** (1999), 5–126 (2000). MR 1793414. Zbl 1005.11030. Available at [http://www.numdam.org/item?id=PMIHES\\_1999\\_\\_89\\_\\_5\\_0](http://www.numdam.org/item?id=PMIHES_1999__89__5_0).
- [SW01] C. M. SKINNER and A. J. WILES, Nearly ordinary deformations of irreducible residual representations, *Ann. Fac. Sci. Toulouse Math. (6)* **10** (2001), no. 1, 185–215. MR 1928993. Zbl 1024.11036. Available at [http://www.numdam.org/item?id=AFST\\_2001\\_6\\_10\\_1\\_185\\_0](http://www.numdam.org/item?id=AFST_2001_6_10_1_185_0).
- [Tay03] R. TAYLOR, On icosahedral Artin representations, II, *Amer. J. Math.* **125** (2003), no. 3, 549–566. MR 1981033. Zbl 1031.11031. doi: 10.1353/ajm.2003.0021.
- [Tay08] R. TAYLOR, Automorphy for some  $l$ -adic lifts of automorphic mod  $l$  Galois representations, II, *Publ. Math. Inst. Hautes Études Sci.* **108** (2008), 183–239. MR 2470688. Zbl 1169.11021. doi: 10.1007/s10240-008-0015-2.
- [Tho12] J. THORNE, On the automorphy of  $l$ -adic Galois representations with small residual image, *J. Inst. Math. Jussieu* **11** (2012), no. 4, 855–920. MR 2979825. Zbl 06101601. doi: 10.1017/S1474748012000023.