

Introduction to
Mathematical Relativity
Sept. 9–13, 2013

Initial Data and Evolution Problems
in General Relativity
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MSRI Workshops

Geometry and Analysis

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